Lectures 12 Temperature and Thermodynamics, Part 2, Concepts

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Topics to be Covered

A. Potential temperature
B. Adiabatic Lapse Rate
C. Thermal Stratification

L12.1 Environmental Measures of Temperature

L12.1.1 Potential Temperature

If a parcel of unit mass **ascends** vertically it **expands**, but its **pressure** also changes. In fact it decreases by *dP*. We should note that this atmospheric case is not like the one with the illustrated situation of a piston of constant mass above the column of air. An atmospheric expansion in the vertical places parcels of air higher into the atmosphere and under a thinner air mass, so from first principles pressure cannot be assumed constant.

On the other hand, we can assume such an expansion is **adiabatic** process. During an adiabatic process matter changes its physical state (either its pressure, volume or temperature change) **without heat** being added or withdrawn, $dQ = 0$.

$$dQ = C_V dT + PdV = 0$$

$$C_V dT = -PdV$$

An adiabatic processes does not infer that **temperature** does not change. This is a major reason why we have introduced the full thermodynamic relation, to understand that there can be a change in temperature with an adiabatic expansion or contraction. Work must be done to compensate, however.

To study adiabatic processes in the atmosphere we normalize the First Law of Thermodynamics mass, m. On the basis of this terminology new variables are introduced:

$$q = \frac{Q}{m}$$
$$w = \frac{W}{m}$$
$$u = \frac{U}{m}$$
$$c_v = \frac{C_v}{m}$$
$$c_p = \frac{C_p}{m}$$

We also define normalized volume in terms of air density, ρ.

$$\frac{dV}{m} = d(\frac{1}{\rho}) = d\alpha$$

An alternative version of *dq* can be derived starting with a definition of an incremental change in the gas law (equation of state).

$$dq = du + Pd\alpha = c_v dT + Pd\alpha$$

It is more convenient to define $Pd\alpha$ in terms of T and P, which are readily measured than $1/\rho$. To do so let's start with an incremental form of the ideal gas law:

$$d(P\alpha) = \frac{1}{m}d(RT) = \alpha dP + Pd\alpha = \frac{1}{m}RdT$$
$$pd\alpha + adp = \frac{R}{m}dT$$

Substituting terms $pd\alpha = -adp + \frac{R}{m}dT$ yields a new for the 1st Law Thermodynamic equation:

$$dq = (c_v + \frac{R}{m})dT - \alpha dp$$

or

$$dq = c_p dT - \alpha dp$$

If we consider an isobaric case where *dp* is zero then we arrive at an alternative definition of the specific heat capacity at constant pressure.

$$\frac{dq}{dT}|_p = c_v + \frac{R}{m} = c_p$$

The specific form of the First Law of Thermodynamics used often in meteorology, as many thermodynamic changes do not involve constant pressure. This form of the equation is used to define adiabatic processes, how temperature changes as we climb a hill and to define potential temperature.

If the ascension is adiabatic, so there is no external supply of heat, the energy for expanding the parcel must come from cooling it, an amount proportional to *dT*.

$$c_p dT - \alpha dp = 0 = c_p dT - \frac{RTdP}{mP}$$

$$\frac{dT}{T} = \frac{R}{m \cdot c_p} \frac{dp}{p}$$

Next integrate both sides

$$\int_\theta^T \frac{dT}{T} = \frac{R}{m \cdot c_p} \int_{p_0}^{p} \frac{dp}{p}$$

and solve for T yields

$$\ln \frac{T}{\theta} = \frac{R}{m \cdot c_p} \ln \frac{p}{p_0}$$

$$\theta = T(\frac{p_0}{p})^{R/(m \cdot c_p)}$$

We, thereby, can quantify the **potential temperature** as the temperature a gas would have if it was expanded or compressed adiabatically to a pressure of 1000mb.

$$\theta = T(\frac{1000}{p})^{R/(m \cdot c_p)}$$

For dry air, $R/m\ C_p$ is 0.286.

We can also use the thermodynamic equation to derive the **adiabatic lapse rate**.

$$dh = C_p dT - \alpha dp$$

$$0 = \rho C_p dT - dp$$

$$\rho C_p dT = dp$$

Taking the derivative with respect to height and substituting the hydrostatic relationship

$$\frac{dp}{dz} = -\rho g$$

yields:

$$\rho C_p \frac{dT}{dz} = \frac{dp}{dz} = -\rho g$$

$$\frac{dT}{dz}\Big|_{adiabatic} = -\frac{g}{C_p} = \Gamma$$

From these equations one can also express the vertical gradient in potential temperature

$$\frac{\partial \theta}{\partial z} = \frac{\theta}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right)$$

If the air is saturated, as when there is fog or clouds, water vapor condenses and latent heat of condensation is released. The moist lapse rate is defined as:

$$\frac{dT}{dz}\Big|_{moist} = -\frac{g}{C_p + \lambda \frac{de_s}{dT}} = \Gamma_{moist}$$

where l is the latent heat of vaporization and de_s/dT is the slope of the saturation vapor pressure-temperature curve.

12.1.5 Adiabatic Lapse Rate

Ever climb a mountain or ridden on a ski lift on a warm day, and by the time you have reached the summit you have to put on a sweater. The reduction of temperature with height is a normal meteorological phenomenon. Because of adiabatic processes, air temperature decreases with height as one climbs a mountain. The dry adiabatic lapse rate, 9.8 K km^{-1}

$$\Gamma_d = -\frac{g}{C_p}$$

The adiabatic lapse rate allows us to define states of thermal stability and instability.

As a first approximation:

$$\theta(z) = T + \frac{g}{c_p} z$$

The atmosphere is neutrally stratified if:

$$\frac{\partial \theta_v}{\partial z} = 0$$

$$\frac{\partial T_v}{\partial z} = -\Gamma$$

The atmosphere is unstably stratified if:

$$\frac{\partial \theta_v}{\partial z} < 0$$

$$\frac{\partial T_v}{\partial z} < -\Gamma$$

In this case if you lift a parcel of air adiabatically it will be **warmer** than the surrounding air and will be **more buoyant**.

The atmosphere is stably stratified if:

$$\frac{\partial \theta_v}{\partial z} > 0$$

$$\frac{\partial T_v}{\partial z} > -\Gamma$$

In this case the atmospheric temperature profile is ‘inverted’. If we lift a parcel of air adiabatically it will be **cooler** than the surrounding air and more dense and **less buoyant**.

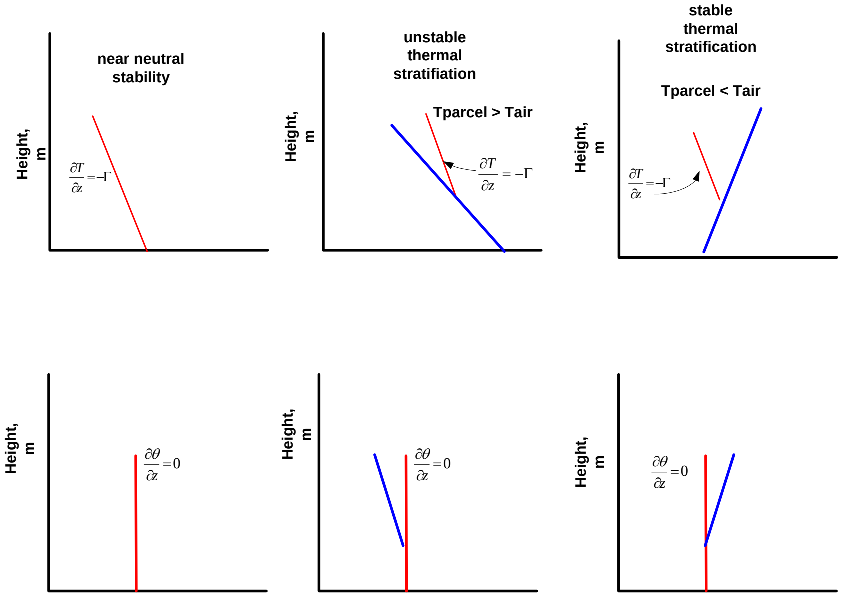

Figure 1 Profiles of temperature and potential temperature for different thermal stratification

In figure 2 is a rawinsonde sounding taken over Oak Ridge, TN during the summer of 1999. One can see regions near the surface of unstable thermal stratification, a region of neutral stratification, capped by an inversion.   These layers are best evident looking at potential virtual temperature instead of dry air temperature, which decreases with height throughout the sounding.

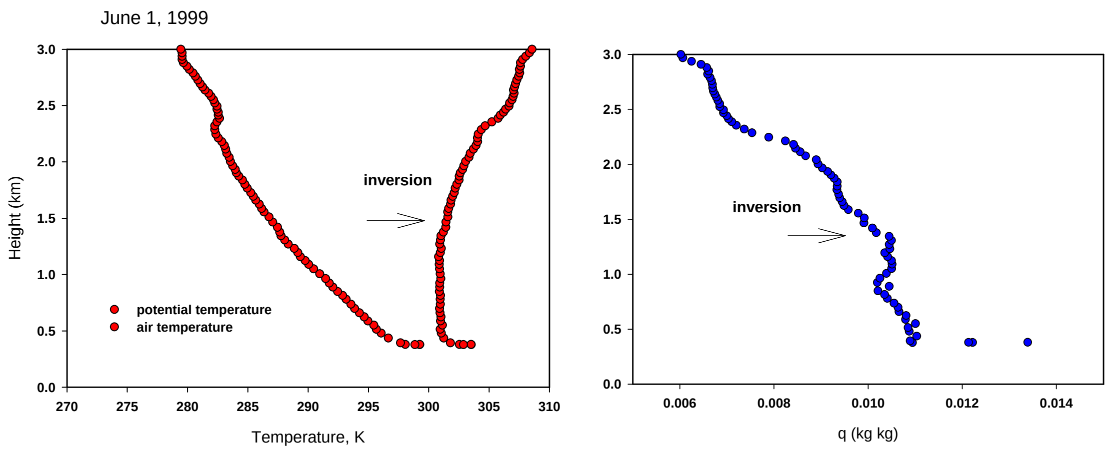

Figure 2 Rawinsonde data over Oak Ridge, TN, June 1, 1999

Summary of Concepts

1. First Law of Thermodynamics

The change in internal energy ΔU is a function of the amount of heat absorbed or lost (ΔQ) and the amount of work done on the system (ΔW)

$$U_2 - U_1 = \Delta U = \Delta Q + \Delta W$$

2. Changes in work are related to changes in volume. At constant pressure:
   a. dW=-P dV
   b. At constant volume, dV is zero, so dW=0 and dU=dQ

3. The Specific Heat at constant pressure is related to the specific heat at constant volume and the Universal Gas Constant, Cp=Cv+R

4. incremental change in heat: $dq = c_p dT - \alpha dp$

5. incremental change in heat: $dq = du + Pd\alpha = c_v dT + Pd\alpha$

6. Potential temperature is the temperature of a parcel of air that is moved adiabatically from a level in the atmosphere to a reference Pressure. It is defined for conditions when changes heat energy are zero (dQ=0).

7. Thermal stratification causes the atmosphere to be either buoyant or stable

The atmosphere is neutrally stratified if: $\frac{\partial \theta_v}{\partial z} = 0$ or $\frac{\partial T_v}{\partial z} = -\Gamma$

The atmosphere is unstably stratified if: $\frac{\partial \theta_v}{\partial z} < 0$ or $\frac{\partial T_v}{\partial z} < -\Gamma$

The atmosphere is stably stratified if: $\frac{\partial \theta_v}{\partial z} > 0$ or $\frac{\partial T_v}{\partial z} > -\Gamma$

3. Biometeorologically important temperatures include air, soil and leaf temperature, and their temperatures on sunny and shaded environments. Other temperatures of note include virtual temperature, potential temperature, aerodynamic temperature, radiative temperature and wet bulb temperature We are also interested in mean temperatures and how they vary over the course of a day and year.

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