Lecture 9 on Integrating or Scaling Information from Leaves to Canopy Scales, Part 3, Multi-Layer Models; Turbulence Closure

ESPM 228, Advanced Topics on Micrometeorology and Biometeorology

Dennis Baldocchi
Professor of Biometeorology
345 Hilgard
University of California, Berkeley
Berkeley, CA
95720
Baldocchi@berkeley.edu

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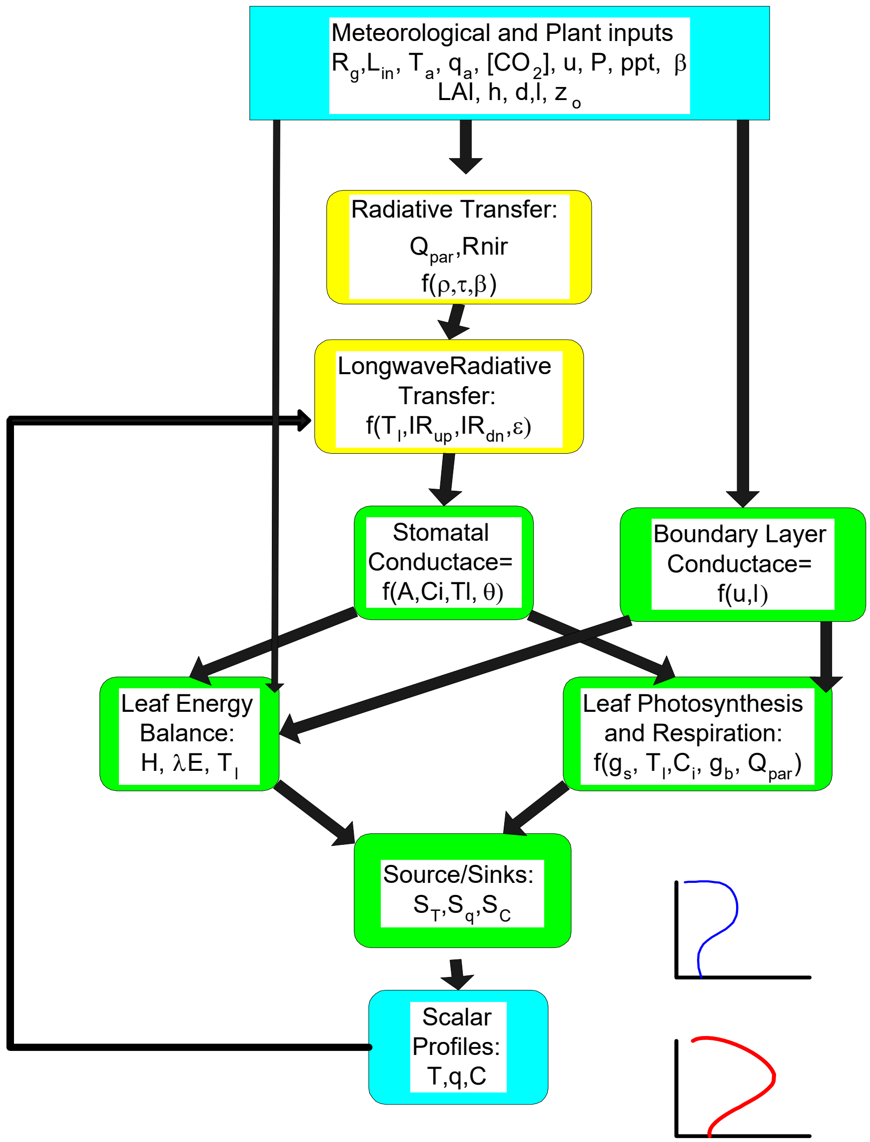
Introduction

Walking through the woods, one readily observed gradients in light, air temperature and canopy structure. Obviously a 'Big-Leaf' model cannot capture the impact of these gradients on mass and energy exchange. Yet, based on our discussion of scaling and integration principles, such heterogeneities can be perceived to be very important, as many trace gas algorithms are non-linear functions of environmental variables that possess non-Gaussian probability distributions. The next level of model complexity in our mathematical "zoo of models" is the multiplayer approach.

Numerous one-dimensional, multi-layer biosphere-atmosphere gas exchange models exist to compute water vapor, CO_2 and isoprene flux densities. Contemporary models consist of coupled micrometeorological and eco-physiological modules. The micrometeorological modules compute leaf and soil energy exchange, wind, momentum transfer, turbulent diffusion, scalar concentration profiles and radiative transfer through the canopy. Environmental variables, computed with the micrometeorological module, in turn, drive the physiological modules that compute leaf photosynthesis, stomatal conductance, transpiration and leaf, bole and soil/root respiration. This information can be used to compute isoprene and monoterpine emission rates, ozone, SO_2, NO_X deposition, as well. We discuss the salient aspects of the model system below.

A schematic of a multi-layer trace gas model with attendant feedbacks is presented below. From this sequence of feedbacks one can speculate how changes in canopy structure, physiology or microclimate may alter other processes.



Starting with the net radiation budget, the interception of sunlight leaves (a function of leaf-sun inclination angle, leaf reflectance and transmissivity) and soil will affect albedo. The amount of radiation available will alter rates of evaporation, sensible and soil heat flux. The amount of leaf area will alter the partitioning of net radiation into H, LE and G. If stomata are relatively closed or the canopy is sparse, surface temperate will be elevated. This effect would enhance longwave loss of radiation and would act to reduce the net radiation budget.

I hope it is becoming evident that it is very difficult to handwave and predict *a priori* which feedbacks dominate and their consequence. Only with the use of a coupled model can we expect to make intelligent and educated guesses on plant-atmosphere interactions at the canopy scale.

I stress this point, for there have been many unexpected results. Leaf physiologists expected stomatal closure, due to elevated CO2 to diminish canopy evaporation and increase water use efficiency. Yet, increases in surface leaf temperature that would be associated with stomatal closure elevated the surface saturation vapor pressure, so LE rates do not diminished, as expected, in direct proportion to stomatal closure. The same is true with drought. One may expect a large reduction in evaporation from potential rates, but one may not observe a large reduction in LE as compared to prior conditions when adequate soil moisture was available.

a. Quantify Sources and Sinks

The conservation budget for a passive scalar provides the foundation for computing scalar fluxes and their local ambient concentrations. If a canopy is horizontally homogeneous and environmental conditions are steady, the scalar conservation equation can be expressed as an equality between the change, with height, of the vertical turbulent flux (*F*) and the diffusive source/sink strength, *S(C,z)*:

$$\frac{\partial F(C,z)}{\partial z} = S(C,z)$$

The diffusive source/sink strength of a scalar in a unit volume of leaves is proportional to the concentration gradient normal to individual leaves, the surface area of individual leaves and the number leaves in the volume. The diffusive source strength can be expressed in the form of a resistance-analog relationship [*Meyers and Paw U*, 1987]:

$$S(C,z) = -a(z)\frac{(C(z) - C_i)}{r_b(z) + r_s(z)}$$

where $a(z)$ is the leaf area density ($m^2\ m^{-3}$), $(C(z) - C_i)$ is the potential difference of scalar concentration or heat content between air outside the laminar boundary layer of leaves and the air within the stomatal cavity ($mol\ mol^{-1}$), r_b is the boundary layer resistance to molecular diffusion ($mol^{-1}\ m^2\ s^1$), and r_s is the stomatal resistance ($mol^{-1}\ m^2\ s^1$).

The previous equation has much information on how plants, soil and the atmosphere interact. Knowledge of a(z) requires information on canopy structure. Ecological principles are needed to understand how a(z) may vary with stand age or functional type [*Parker*, 1995].

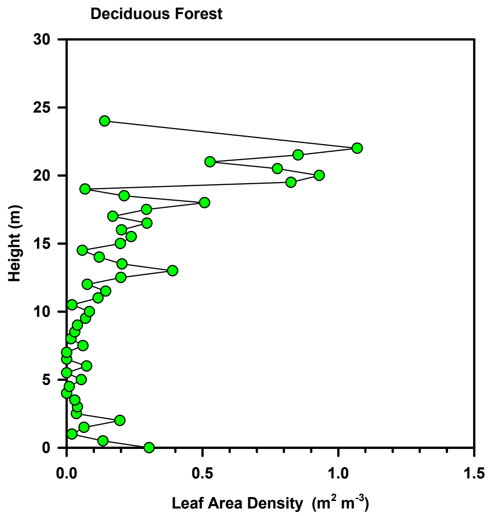


Figure 1 Example of vertical profile of leaf area index, after Hutchison et al. 1983. These data are fore a 40 year old temperate broadleaved forest.

Leaf boundary layer resistances for molecular compounds can be computed using flat plate theory [*Schuepp*, 1993]. In principle such resistances, under forced convection, are a function of a leaf's length scale (*l*), molecular diffusivity (*d*) and the Sherwood number, *Sh*.

$$r_b = \frac{l}{d \cdot Sh}$$

The boundary layer resistance requires information on wind speed, turbulence, leaf size and sheltering. Yet to understand how wind speed diminishes in a canopy we must understand momentum transfer, which is also affected by leaf area index and its distribution.

The stomatal resistances is one of the most prominent physiological terms. It is a function of plant functional type, physiological status, photosynthetic pathway, light, temperature and humidity exposure.

Now we start to see how more and more microclimate information is needed to understand the behavior of the stand. The internal concentration generally is a biochemical or physicochemical property. For example, a set of biochemical reactions for photosynthesis results in the set point for Ci, for CO_2. Whereas if we are interested in ozone we may rely on Henry's Law. For water vapor we need the Clausius-Clapeyron equation to compute the interstomatal concentration, Ci. Finally we come back to C, the concentration in the air space. If the air is well mixed and the source –sinks are weak C may be relatively constant in the canopy. Strong gradients in C can occur within a canopy when source/sink strengths are strong and turbulence is weak. Counter-gradient transfer occurs with turbulent transport (as denoted by the third order moment) is significant:

$$\frac{\partial c}{\partial z}=[\frac{\overline{w'c'}}{\tau}-\frac{\overline{\partial w'w'c'}}{\partial z}]/\overline{w'w'}$$

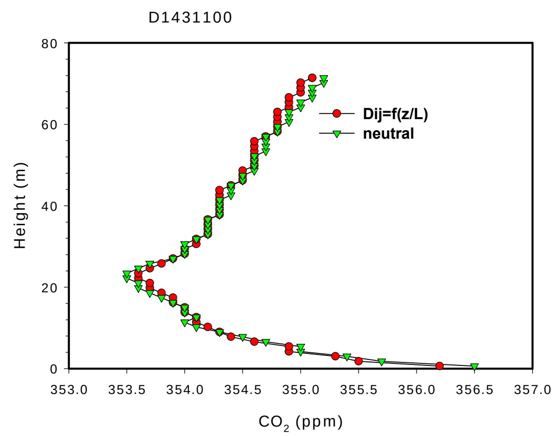


Figure 2. Model calculations showing theoretical sensitivity of CO2 profiles to thermal instability above the canopy. Z/L is xxx.

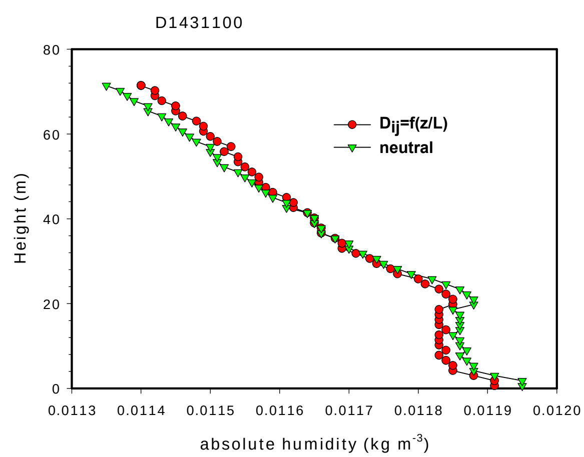


Figure 3. theoretical computations of humidity profiles assuming near neutral and unstable thermal stratification above the canopy.

Chemical reactions are important when the time scale of the reactions are shorter than the turbulence time scale that determines the residence time of a parcel of air (Gao et al., 1993). In this case the source/sink term is expanded to include chemical production and destruction (S_{ch}):

$$\frac{\partial F(c,z)}{\partial z} = S_B(c,z) + S_{ch}(c,z)$$

In the simplest circumstance, S_{ch} is parameterized using chemical kinetics, where the rate of reaction is proportional to the local concentration:

$$S_{ch} = -kc(z)$$

The introduction of chemistry into a canopy trace gas exchange model increases the need to compute scalar profiles accurately. This is because errors attributed to the parameterization of turbulence and scalar profiles will translate directly into errors in the evaluation of chemical kinetics. The other issue associated with the evaluation of source/sink Equation involves what suite of chemical compounds to consider. Photochemical models tend to involve hundreds of reactions, which can be reduced to a suite of 20 to 40 key reactions.

Computing Turbulence and Concentration Profiles, the Need for Closure Schemes

The conservation budget equation for a scalar cannot be readily solved because it does not form a closed set of equations and unknowns. The equation defining the time rate of change in c contains a higher order moment, which is also a function of c. This higher order moment is the vertical turbulent flux (F), which is defined as the covariance between vertical velocity (w) and scalar concentration fluctuations ($\overline{w'c'}$) (primes denote fluctuations from the mean and the overbar represents time averaging). The exercise of closure involves defining an equation of set of equations that define the highest order term as a function of lower order moments. Table 1 shows how the closure problem cascades in complexity as one progress from zero order closure to third order closure. More and more unknowns come in existence, as due the number of equations that require solving.

Table on fluid dynamic Equations and Closure

Order of Closure	**Turbulence Budget Equations**	**Unknowns**	**Scalar Budget Equations** $(\frac{\partial c}{\partial t})$	**Unknowns**	**Closure Scheme**
zero		$\overline{u}, \overline{w}$		$\overline{T}, \overline{q}, \overline{C}$	$\overline{c} = f(t,x,y,z)$
First Order	$\overline{u}$	$\overline{u'u'}, \overline{w'w'}, \overline{w'u}$	$\overline{T}, \overline{q}, \overline{C}$	$\overline{T'^2}, \overline{q'^2}, \overline{C'^2}, \overline{w'}$ $\overline{w'q'}, \overline{w'C'}$	$\overline{w'c'} = -K\frac{\partial c}{\partial z}$

Second Order	$\overline{u'u'},\overline{w'w'},\overline{w'}$ $\overline{v'v'}$	$\overline{u'u'u'},\overline{w'w'w}$ $\overline{w'u'u'},\overline{u'w'w}$	$\overline{T'^2},\overline{q'^2},\overline{w'T'}$ $\overline{w'q'},\overline{T'q'},\overline{q'}$	Third order moments	?? $\overline{w'w'c'}=-K\frac{\partial\overline{w'c'}}{\partial z}$
Third Order	$\overline{w'w'u'},\overline{w'w'}$ $\overline{w'u'u'},\overline{w'v'v}$		$\overline{w'w'T'},\overline{w'T}$ $\overline{u'w'T'},\overline{w'T'}$ $\overline{w'w'q'}$	Fourth order moments	$\overline{w'w'w'w'}=$ $3\overline{w'w'}\cdot\overline{w'w'}$ quasi-Gaussian approximation

Two basic reference frames exist for evaluating the conservation budget. They are the Eulerian and Lagrangian frames. The Eulerian framework describes how the scalar concentration varies with time (t) at a fixed point in space (x,y,z). It is equivalent to measuring the concentration of a given scalar from a tower. Current modeling approaches apply statistical principles to physical relationships for scalar and wind conservation to develop a set of equations that predict the spatial behavior of statistical moments, e.g .means, variance and covariances of turbulence quantities. Such models do not predict the behavior of arbitrary plumes or fluid particles. When one derives a budget equation for a second order moment, it becomes readily evident that a third order moment term arises, and so on.

The Lagrangian approach analyzes the conservation equation by following parcels of fluid as they move with the wind, much like the trajectory of a neutrally-buoyant balloon. This approach simulates changes in motion with a statistical-mechanical approach. Movement is partly a function of persistence and of random forcings.

The principles behind these frameworks are explored in the following subsections. Each method has distinct advantages and disadvantages over the other. But none circumvent the closure problem. For instance, Eulerian models rely on some type of K theory closure, even if it is restricted to a higher moment [*J. W. Deardorff*, 1978]. Lagrangian models capture the essences of counter-gradient diffusion, but there 'closure' is achieved by prescribing the attributes of turbulence, rather than predicting them. It leaves the solution of the equations of fluid mechanics one of the great unsolved problems of physics.

1. Eulerian Models
2.
    a. Zero Order Closure

The simplest closure scheme is to assume that the concentration field inside a plant canopy is invariant with height (or varies with some prescribed relationship). This is an assumption that has been adopted by many early ecological models. They assume air temperature, humidity, and CO_2

concentration is the same value inside the canopy as at some reference level. This approach has been applied often to drive photosynthesis models.

One use of the validated and detailed, micrometeorology model is as a tool for guiding the development of simpler parameterization schemes. For example, by inter-comparing flux measurements with a detailed canopy micrometeorology model and a simpler closure scheme one can evaluate how the ability to estimate fluxes changes with changing model sophistication. Theoretically, source-sink strengths are dependent on the scalar field and vice versa. If strong gradients exist inside a plant canopy and the source-sink term is sensitive to the scalar concentration, it can be argued that a micrometeorology model, which resolves the local concentration field, is required to calculate fluxes. On the other hand, one could argue that simple half-order closure models would be useful if the scalar concentration profile is uniform or if the source-sink parameterization is insensitive to changes in scalar quantity.

To test whether we need a simple or complex parameterization scheme for calculating sensible heat exchange (H) let's first investigate how sensitive H is to observed drawdowns or build-ups in air temperature (T_a). We can define this sensitivity by using elementary calculus to determine the partial derivative of H with respect to air temperature:

$$\frac{\partial H}{\partial T} = \frac{2a(z)\rho_a C_p}{r_a}$$

where C_p is the specific heat of air. Assuming a(z) equals 1 m^{-1} and r_a equals 20 s m^{-1}, yields a sensitivity of about -120 W m^{-2} C^{-1}. Is this sensitivity of H to air temperature strong enough to hinder the computation of H by ignoring vertical variations in the temperature profile? We can answer this question by comparing measurements of H against model computations which assume that air temperature constant and varies with height. Agreement between measured and calculated sensible heat flux densities is poorer when the model assumes that air temperatures are constant with height rather than when air temperatures vary with height. For example, the half-order closure scheme underestimates measured values by 41 W m^{-2}, while the integrated canopy model underestimates measured H values by only 14 W m^{-2}. A conclusion that can be drawn from the above discussion is that use of the half-order closure model scheme is ill-advised for the computation of H.

What about using a half-order closure model to compute LE and F_c? Again, let's examine how sensitive computations of these flux densities are to their respective scalar. The sensitivity of LE to changes in humidity can be calculated from the partial derivative of LE with respect to Δ_v:

$$\frac{\partial \lambda E}{\partial \rho_v} = \frac{\lambda a(z)}{r_a + r_s}$$

where λ is the latent heat of evaporation. Assuming a(z) equals 1 m^{-1}, r_a equals 20 s m^{-1} and r_s equals 50 s m^{-1}, yields a sensitivity value of -35 W m^{-2} (g m^{-3})$^{-1}$. The sensitivity of LE to Δ_v,

$\partial LE/\partial\Delta_v$, is much less than the sensitivity of H to T_a, $\partial H/\partial T_a$. This reduction in sensitivity is brought about by control of stomata on vapor diffusion through leaves.

Does the sensitivity of LE to humidity hinder computations of LE that disregard vertical variations in the humidity profile? We can address this question by comparing measurements of LE against model computations which assume that the humidity is constant and varies with height. Here, we find that computations of LE based on a constant humidity profile is well correlated with measured values (r^2=0.99; slope=1.03; Table 3) and differ measured values by only 21 W m^{-2}. Considering that LE values during midday range between 400 and 600 W m^{-2}, a 21 W m^{-2} difference is trivial during this period. Furthermore, the performance of the half-order closure model is not much worse than the performance of the integrated Lagrangian model, which differed from measured values by 12 W m^{-2}. Based on these calculations, it can be concluded that a simple half-order parameterization scheme can be used to calculate LE under certain conditions, the scheme will work best when r_s is much greater than r_b.

The influence of assuming a constant CO_2 profile on the computation of canopy CO_2 exchange has been addressed in another analysis [*Baldocchi*, 1992]. It was shown for soybeans and a deciduous forest that computations of F_c are relatively insensitive to variations in the [CO_2] profile, so a half-order closure scheme is valid for modelling this variable too.

More recently we updated this exercise for a deciduous forest and examined the impact of assuming zero order closure with regards to daily integrals of mass and energy exchange. The following figure shows that errors are not too great for Fc and LE. On the other hand substantial errors occur with regards to computing H if T profiles are ignored. Please recognize, H is a major component of computing PBL growth, the generation of convective clouds and precipitation. Ignoring this important feedback could have dire consequences on the computation of weather and climate.

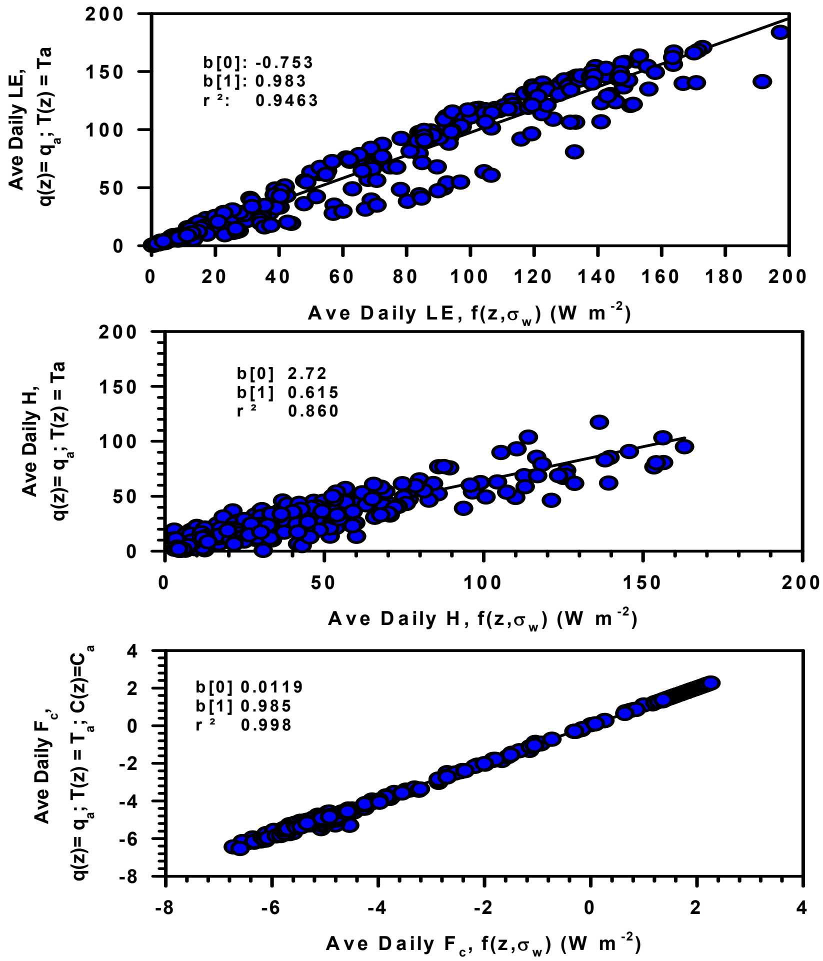


Figure 4 Calculations of mass and energy exchange by either considering feedbacks between sources/sinks and local scalar concentration fields or by assuming that the profiles are invariant and equal to the above canopy value.

b. First Order Closure

In our discussion of sources and sinks we have observed that the source/sink term is a function of the local concentration field and that the local concentration field is a function of the source/sink strength and turbulent diffusion. The simplest (and earliest) Eulerian models on turbulent exchange in plant canopies adopted a first order closure scheme, called 'K-theory' [*J Finnigan*, 2000; *M. R. Raupach and Thom*, 1981]. The appeal of this model is in its simple reduction of the number of unknown variables. K-theory models assume that turbulent transfer and molecular diffusion are analogs, thereby the vertical velocity-scalar covariance is represented as the product of the scalar concentration gradient and a turbulent diffusivity (K): $F_c(z) = \overline{w'c'} = -K\frac{\partial \bar{c}}{\partial z}$. The rationale for this approach, at the time, was based on a presumption that the scales of mixing were small, as leaves broke eddies down to smaller and smaller sizes. Without the aid of eddy covariance measurements, early experimentalists and modelers were unaware of how non-local transport affected local exchange. Nor were they aware of the fact that the mean length scale of turbulent transfer was on the order of canopy height rather than the length scale of leaves.

To arrive at an algorithm for computing eddy exchange coefficients inside a canopy, it was common to assume Reynolds analogy where K_m=K_v=K_h and to compute Km from information on leaf area density and wind speed profiles and drag coefficients [*M. R. Raupach and Thom*, 1981].

$$K_m(z) = \frac{\tau(z)}{\rho\, \partial \bar{u} / \partial z}$$

$$\overline{u(z)} = u_h \exp(-\alpha(1-\frac{z}{h}))$$

$$\tau(z) = \int_z^h u(z)^2 a(z) C_d(z) dz$$

Another approach was to use the energy balance method to assign values for K

$$K(z) = \frac{R_n(z)}{\overline{\rho_a}(C_p \frac{\partial \theta(z)}{\partial z} + \frac{\varepsilon}{P}\lambda \frac{\partial e(z)}{\partial z})}$$

This method would need linearly averaged measurements of net radiation in the canopy and arrays of temperature and humidity sensors. The drawback of this approach is the placement, size and aspiration of such sensors in short-statured crops. In tall forests, it is nearly impossible to measure spatially averaged values of net radiation without some elaborate tram system.

A third approach for K comes from the shear production term of the TKE budget:

$$K_m(z) \approx \tau_l(z)\sigma_w^2(z) \text{ (m}^2\text{ s}^{-1}\text{)}$$

As a first approximation, one can use the 'Family Portrait" of turbulence profiles that have been assembled by Raupach and co-workers.

In a classic paper by the fluid dynamics pioneer, Stanley Corrsin [*Corrsin*, 1974], he wrote that several conditions must hold to apply K-theory. First of all, the length scales of the turbulent transfer must be less than the length scales associated with the curvature of the concentration gradient of the scalar. Secondly, the turbulent length scale must be constant over the distance where the concentration gradient changes significantly.

K-theory models were originally thought to be valid because many presumed that turbulence was produced in the wakes of the foliage. On this assumption, turbulent length scales were assumed to be sufficiently small to comply with Corrsin's [*Corrsin*, 1974] restrictions.

An accumulating body of evidence now shows that these prior assumptions are often not valid inside plant canopies. Much turbulent transfer is associated with coherent and intermittent wind gusts, whose length scales are comparable to or greater than the vegetation height [*M.R. Raupach*, 1988]. Since concentration gradients of many scalars exhibit strong curvature due to the local contribution of its source the length scale that represents the curvature of the scalar profile will be less than that for turbulence, violating one of Corrin's rules. Proof that first order K-theory can be invalid inside plant canopies faced its death knell from observations of counter-gradient transfer of heat and momentum ([*Baldocchi and Meyers*, 1988; *Denmead and Bradley*, 1987] the diffusion analogy cannot admit negative values for K, as would otherwise occur under such circumstances (see Thurtell, 1989). Raupach [*M. R. Raupach*, 1987] explains counter-gradient transfer with the following explanation:

> *'(because) scalar from nearby elementary sources is dispersing in a near-field regime...its contribution to the overall **gradient** (is) much greater than its contribution to the overall **flux density**. Just below a fairly localized and intense source in the canopy, the near-field gradient contribution is large and positive; when this is combined with the upward flux of scalar required by conservation of scalar mass, a countergradient flux is obtained'.*

Since K-theory is invalid inside a plant canopy one cannot derive canopy-level exchange rates from concentration gradients measured inside the canopy, as was unsuspectingly discovered by Johnson et al. (1976).

Another way to demonstrate why K theory does not work in plant canopies is to use the simplified version of the conservation budget equation for the eddy covariance (as applied by [*Wyngaard*, 1992].

$$\overline{w'c'} = \tau(\overline{w'^2}\frac{\partial \overline{c}}{\partial z} + \frac{\partial \overline{w'w'c'}}{\partial z})$$

This equation can be represented as the sum of a flux-gradient term and a term that arises from non-local turbulent transport (TT), the flux divergence of the flux of the scalar flux:

$$\overline{w'c'} \sim K_c \frac{\partial \overline{c}}{\partial z} + TT$$

Inside a canopy, the vertical gradient of the third order moment is non-zero because large eddies are transporting material to and from the local area. This non-local transport is a direct cause of the failure of K theory. In contrast this TT term is near zero above a canopy in the internal boundary layer.

b. Second Order Closure Models

There are several unique features of canopy turbulence that must be accounted for by a competent theory:

1. momentum is absorbed by the ground and elevated plant elements
2. a hydrodynamically unstable inflexion occurs in the wind profile at the canopy atmospheric interface.
3. Turbulence in the vegetation is vertically inhomogeneous.
4. mean kinetic energy is converted to turbulent kinetic energy in the wake of plants, which accelerates the eddy cascade.

Higher-order closure models have been proposed as a means of circumventing the inherent limitation of first order closure models. The appeal of this method is its mechanistic base and an ability to simulate counter-gradient transport [*N R Wilson and Shaw*, 1977]. Higher order closure models introduce formal budget equations for higher order moments, such as $\overline{w'c'}$ or $\overline{w'u'}$.

Wilson and Shaw [*N R Wilson and Shaw*, 1977]produced the first higher order closure model for canopy flow. The concept has earlier origins in surface layer micrometeorology (duPont Donaldson, 1975; [*Wyngaard*, 1992; *Wyngaard and Cote*, 1971] and engineering flows (Launder et al., 1975). The next step in development involved the derivation of the closure equations for volume averaging by Raupach and Shaw [*M. R. Raupach and Shaw*, 1982] and Finnigan ([*J J Finnigan*, 1985].

> *"solid plant parts are excluded from the averaging volume so that the averaging proceeds over a multiply connected space and source or sink terms appear as the sums o fluxes across the solid boundaries internal to the averaging volume. In horizontally homogeneous canopies the choice of averaging is usually a thin wind horizontal slab that preserves the fundamental vertical heterogeneity of the canopy but reflects its horizontal uniformity on the scale of many plants" [Ayotte et al., 1999].*

Finnigan [*J J Finnigan*, 1985]has also derived equations that contain terms for wake production and waving plants.

In the mid 1980s Meyers and Paw U [*Meyers and Paw U*, 1987]developed a third order closure scheme to improve the parameterization of large scale turbulence and pressure transport terms, as was being observed by field experiments. Their aim was to remedy current schemes that used an effective K for computing transport. They used quasi-Gaussian approximation to represent the fourth order moments (they were approximated at combinations of second order terms).

Wilson [*J D Wilson*, 1988] built on ideas from field studies that were showing a short circuiting of the inertial cascade. He divided the TKE budget into two different scales, the larger scale, shear kinetic energy, and the smaller, wake kinetic energy. In this form loss of SKE is a gain by WKE. He parameterized the dissipation shear kinetic energy (SKE) according to work of turbulence against form drag (which short-circuited the eddy cascade) and conventional viscous dissipation. Most recent additions to higher order closure theory have been papers by Katul and Albertson [*Katul and Albertson*, 1999]), Ayotte et al., [*Ayotte et al.*, 1999]and Massman and Weil [*Massman and Weil*, 1999].

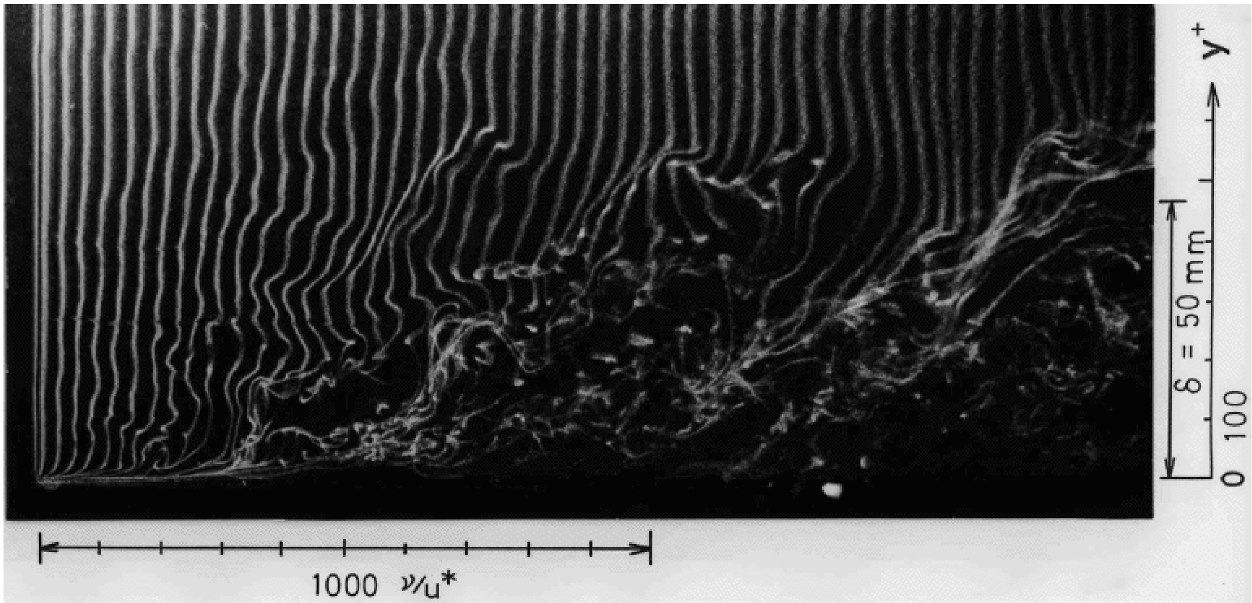
Equations that describe mean wind speed and turbulence are introduced to evaluate dependent terms in the second moment equation and in the source-sink function (i.e. r_b and c(z)). For example, wind speed and turbulence in a plant canopy are described by the budget equations for mean horizontal wind velocity ($\overline{u}$), tangential momentum stress ($\overline{w'u'}$) and the turbulent kinetic energy components ($\overline{u'u'}, \overline{v'v'}, \overline{w'w'}$).

To understand the closure problem and the 'mathematical zoo' of equations that evolve, let's start with the 'simplest' example of the higher order closure problem, how to assess the budget equation for wind velocity, u ([*N R Wilson and Shaw*, 1977]). To solve this problem one starts with the Navier-Stokes equation for the conservation equation for a wind velocity component. The Navier-Stokes equation states that the time rate of change of wind velocity (acceleration) is caused by the foreces that are imposed on the fluid. These forces are associated with pressure gradients, thermal buoyancy and viscous drag. Using tensor notation, this equation is expressed as:

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{g\theta}{T}\delta_{i3} + \upsilon \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

The physical representation of this equation is rather simple. We note that in the atmospheric surface layer and plant canopy, we ignore the Coriolis force.

The problem we face arises with solving the equation as turbulence is multiscaled. Turbulence shows behavior that is both large scale and coherent and fine scale and chaotic. This behavior arises in part from the non-linear nature of the Navier-Stokes equation ($\frac{\partial u}{\partial t} = f(u \cdot u)$)



from Turbulence and Heat Transfer Laboratory, University of Toyko

http://www.thtlab.t.u-tokyo.ac.jp/index.html

In deriving the set of guiding equations, we use Einstein tensor notation, u_1,u_2,u_3 corresponds with the vectors u,v,w which are directed in the longitudinally (x), lateral (y) and vertical (z) directions. Summation is implied when there is a repeated index, eg

$A_i = a_{ij} A_j = a_{i1} A_1 + a_{i2} A_2 + a_{i3} A_3$ or

$$u_j \frac{\partial u_i}{\partial x_j} = u_1 \frac{\partial u_i}{\partial x_1} + u_2 \frac{\partial u_i}{\partial x_2} + u_3 \frac{\partial u_i}{\partial x_3}$$

On the basis of this equation, one can derive mean equations for conservation budget of momentum transfer and turbulent kinetic energy (or the variance equations of the three orthogonal wind velocities), using rules of Reynolds' averaging. Ultimately one needs to solve a system of equations that contains 5 equations and 5 unknowns.

The time-averaged version of du/dt leads to:

$$\frac{\overline{du(t,x,y,z)}}{dt} = \frac{\partial \overline{u_i}}{\partial t} + \overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_i' u_j'}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \overline{p'}}{\partial x_i} + \frac{g \overline{\theta}}{T} \delta_{i3} + \upsilon \frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j} + \upsilon \frac{\partial^2 \overline{u_i'}}{\partial x_j \partial x_j}$$

The application, of several rules of Calculus, lead to the non-linear second order terms. First the total time derivative can be expressed as the sum of the local time derivative and the 'advection' terms:

$$\frac{dc(t,x,y,z)}{dt} =$$

$$\frac{\partial c}{\partial t} + \frac{dx}{dt}\frac{\partial c}{\partial x} + \frac{dy}{dt}\frac{\partial c}{\partial y} + \frac{dz}{dt}\frac{\partial c}{\partial z} =$$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z}$$

Term 1, the time derivative:

$$\overline{\frac{\partial u_i}{\partial t}} = \overline{\frac{\partial(\bar{u} + u')}{\partial t}} = \overline{\frac{\partial \bar{u}}{\partial t} + \frac{\partial u'}{\partial t}} = \frac{\partial \bar{u}}{\partial t}$$

Term 2, the advection term:

$$\overline{u_j \frac{\partial u_i}{\partial x_j}} = \overline{(\bar{u} + u')}\overline{(\frac{\partial \bar{u}}{\partial x} + \frac{\partial u'}{\partial x})} =$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \overline{u'\frac{\partial \bar{u}}{\partial x}} + \overline{\bar{u}\frac{\partial u'}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}}$$

Term 3: the pressure gradient term:

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial \bar{p}}{\partial x_i} + \overline{\frac{\partial p'}{\partial x_i}}$$

Term 4, viscous drag term:

$$\overline{\upsilon \frac{\partial^2 u}{\partial x_j \partial x_j}} = \upsilon \frac{\partial^2 \bar{u}}{\partial x_j \partial x_j} + \overline{\upsilon \frac{\partial^2 u'}{\partial x_j \partial x_j}}$$

Time averaging $u_j \frac{\partial u_i}{\partial x_j}$ results in the production of a mean advection term, $\bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j}$, and a second order covariance term $\frac{\partial \overline{u_i' u_j'}}{\partial x_j}$. This covariance term is the basis behind higher order closure theory and problem. The production of a mean budget equation at a given order by time averaging forms a higher order term. Its form is based on a mathematical trick and use of knowledge about the behavior of incompressible fluids. When one time averages the instantaneous NS equation we arrive with one term

$$\frac{\overline{u_j'\partial u_i'}}{\partial x_j}$$

We know that for incompressible fluids: $\frac{\partial u_j'}{\partial x_j} = 0 = \frac{\partial u_j}{\partial x_j} = \frac{\partial \overline{u_j}}{\partial x_j}$

$$\frac{\partial \overline{u_i' u_j'}}{\partial x_j} = \overline{u_i' \frac{\partial u_j'}{\partial x_j}} + \overline{u_j' \frac{\partial u_i'}{\partial x_j}}$$

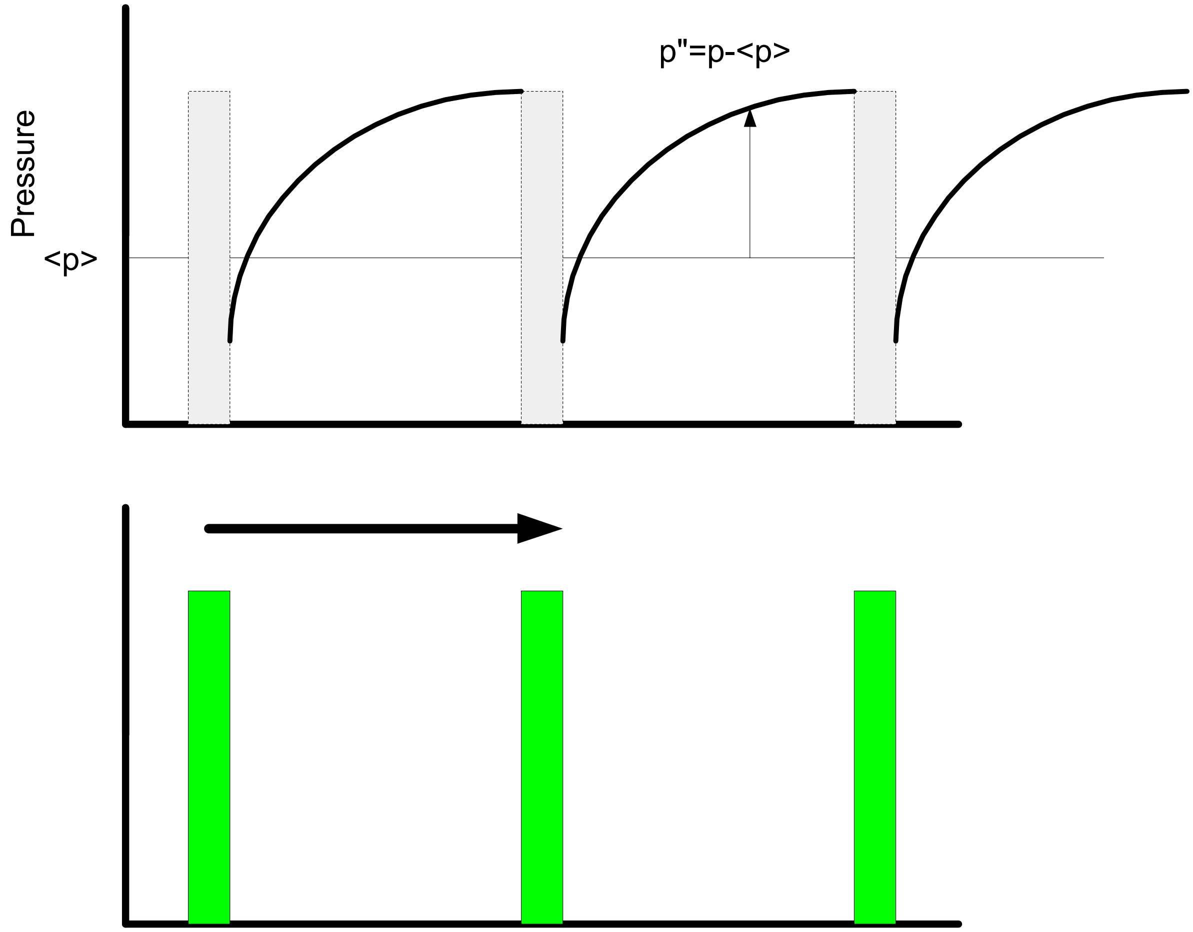
yielding

$$\frac{\partial \overline{u_i' u_j'}}{\partial x_j} = \overline{u_j' \frac{\partial u_i'}{\partial x_j}}$$

Contrasting canopy and surface layer flow reveals the relative importance of different terms.

In atmospheric flow the second and fifth terms on the RHS are normally negligible. Inside a plant canopy, commonly neglected terms are important and significant. The cited terms represent form (pressure) drag and viscous drag.

Form drag is due to pressure gradients across solid obstructions to the wind field.



We note the average gradient of pressure fluctuations is non-zero, which the gradient of averaged pressure fluctuations is zero: $\overline{\frac{\partial p'}{\partial x}} \neq \frac{\partial \overline{p'}}{\partial x} = 0$. One can see pressure deficits in the lee of wind obstructions.

Viscous drag term arises because terms like $\frac{\partial u'}{\partial z}$ are non-zero as the viscous drag over leaf surfaces is distributed in space.

In the case of fluid flow in a canopy the following the set of equations needed to be solved.

1. *Horizontal velocity*

$$\frac{\partial \overline{u}}{\partial t} = 0 = -\frac{\partial \overline{u'w'}}{\partial z} - \frac{\overline{\partial p'}}{\rho \partial x} - \frac{\partial \overline{p}}{\rho \partial x} + \upsilon \frac{\partial^2 \overline{u}}{\partial z \partial z}$$

Wilson and Shaw (1977), among others, assume that the pressure fluctuation gradient can be assessed as a drag force:

$$\frac{\overline{\partial p'}}{\partial x_i} = C_d a(z)\overline{u_i}^2$$

They also assume that the viscous drag force and mean horizontal pressure gradients are small in comparison. This assumption, for a horizontally homogeneous canopy exposed to steady state conditions, leads to a balance between the flux divergence of the Reynolds stress and the drag force:

$$\frac{\partial \overline{w'u'}}{\partial z} = C_d a(z)\overline{u}^2$$

Inside a canopy, the flux divergence of momentum transfer is a function of a drag coefficient, leaf area density and velocity squared. This term is quite distinct from its value above a canopy, which is zero. Or in the plantetary boundary layer where pressure gradients and Coriolis forces come into play (and lead to the Eckmann spiral).

2. *Second Order Moments*

Multiplying u_i times du'/dt and applying rules of Reynolds averaging produces budget equations for second order moments, uw, Reynolds shear stress and uu,vv and ww, components of the turbulent kinetic energy equation.

The general equation for second order velocity components is:

$$\frac{\partial \overline{u_i'u_k'}}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i'u_k'}}{\partial x_j} + \overline{u_k'u_j'}\frac{\partial \overline{u_i}}{\partial x_j} + \overline{u_i'u_j'}\frac{\partial \overline{u_k}}{\partial x_j} + \frac{\partial \overline{u_i'u_k'u_j'}}{\partial x_j} =$$

$$-\left[\frac{\partial \overline{u_k'p'}}{\partial x_i} + \frac{\partial \overline{u_i'p'}}{\partial x_k}\right] + \left[\overline{u_k}\frac{\partial \overline{p'}}{\partial x_i} + \overline{u_i}\frac{\partial \overline{p'}}{\partial x_k}\right] + \overline{p'\left[\frac{\partial u_k'}{\partial x_i} + \frac{\partial u_i'}{\partial x_k}\right]} + \frac{g}{T}\left[\overline{\theta' u_i'}\delta_{k3} + \overline{\theta' u_k'}\delta_{i3}\right] + \upsilon\frac{\partial^2 \overline{u_i'u_k'}}{\partial x_j \partial x_j} - 2\upsilon\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_k'}{\partial x_j}}$$

From this long and complicated equation we can compute time averaged equations for components of the turbulent kinetic energy budget (variance) and momentum transfer [*Meyers and Paw U*, 1987; *Shaw*, 1977]

$$\frac{\partial \overline{u'u'}}{\partial t} = 0 = -2\overline{w'u'}\frac{\partial \overline{u}}{\partial z} - \frac{\partial \overline{w'u'u'}}{\partial z} - 2\frac{\overline{u'\partial p'}}{\rho \partial x} - 2\frac{\overline{u}\overline{\partial p'}}{\rho \partial x} - \frac{2}{3}\varepsilon$$

$$\frac{\partial \overline{v'v'}}{\partial t} = 0 = -\frac{\partial \overline{w'v'v'}}{\partial z} - 2\frac{\overline{v'\partial p'}}{\rho \partial y} - \frac{2}{3}\varepsilon$$
$$\frac{\partial \overline{w'w'}}{\partial t} = 0 = -\frac{\partial \overline{w'w'w'}}{\partial z} - 2\frac{\overline{w'\partial p'}}{\rho \partial z} - 2\frac{g}{\theta}\overline{w'\theta_v'} - \frac{2}{3}\varepsilon$$

A simplified version of the total tke ($\bar{e} = \frac{1}{2}\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right)$) budget is:

$$\frac{\partial \bar{e}}{\partial t} = 0 = -\overline{w'u'}\frac{\partial \bar{u}}{\partial z} + \frac{g\overline{w'\theta'}}{\bar{\theta}} - \frac{\partial \overline{w'e'}}{\partial z} - \frac{\partial \overline{p'w'}}{\rho \partial z} - \varepsilon$$

Note some pressure terms are missing from the integral equation (check::: because of return to isotropy, the summation of the pressure terms is zero.

$$\varepsilon \sim \frac{u_*^3}{kz}$$

Momentum Budget

$$\frac{\partial \overline{u'w'}}{\partial t} = 0 = -\overline{w'w'}\frac{\partial \bar{u}}{\partial z} - \frac{\partial \overline{w'u'u'}}{\partial z} + \left(\frac{\overline{p'\partial u'}}{\partial z} + \frac{\overline{p'\partial w'}}{\partial x}\right)$$

When one applies time averaging, no additional terms are added to the momentum budget equation (as compared to the free atmosphere) when it is applied to a vegetated canopy [*Shaw*, 1977]. Shaw (1977) goes on to defines the terms on the right hand side as: I) shear production due to interactions between the turbulent field and the mean velocity gradient, II) net transport of stress and III) is a return to isotropy term due to pressure and velocity interactions. With

Scalar conservation, variance and co-variance equations

$$\frac{\partial \bar{\theta}}{\partial t} = 0 = -\frac{\partial \overline{w'\theta'}}{\partial z} + S_\theta$$
$$\frac{\partial \bar{q}}{\partial t} = 0 = -\frac{\partial \overline{w'q'}}{\partial z} + S_q$$
$$\frac{\partial \overline{w'\theta'}}{\partial t} = 0 = -\overline{w'w'}\frac{\partial \bar{\theta}}{\partial z} - \frac{\partial \overline{w'w'\theta'}}{\partial z} - \frac{\overline{\theta'\partial p'}}{\partial z} + g\beta\overline{\theta_v'^2}$$
$$\frac{\partial \overline{w'q'}}{\partial t} = 0 = -\overline{w'w'}\frac{\partial \bar{q}}{\partial z} - \frac{\partial \overline{w'w'q'}}{\partial z} - \frac{\overline{q'\partial p'}}{\partial z} + g\beta\overline{\theta'_v q'}$$

$$\frac{\partial \overline{u'\theta'}}{\partial t} = 0 = -\overline{u'w'}\frac{\partial \overline{\theta}}{\partial z} - \overline{w'\theta'}\frac{\partial \overline{u}}{\partial z} - \frac{\partial \overline{w'u'\theta'}}{\partial z} - \frac{\overline{\theta'\partial p'}}{\partial x}$$

$$\frac{\partial \overline{\theta' q'}}{\partial t} = 0 = -\overline{w'q'}\frac{\partial \overline{\theta}}{\partial z} - \overline{w'\theta'}\frac{\partial \overline{q}}{\partial z} - \frac{\partial \overline{w'q'\theta'}}{\partial z} - 2\varepsilon_{\theta q}$$

$$\frac{\partial \overline{\theta'\theta'}}{\partial t} = 0 = -2\overline{w'\theta'}\frac{\partial \overline{\theta}}{\partial z} - \frac{\partial \overline{w'\theta'\theta'}}{\partial z} - \varepsilon_{\theta\theta}$$

Horizontally Averaged Equations

The equation presented so far are for a point in space. Due to the structural heterogeneity of plant canopies, many canopy micrometeorologist now prefer to present the conservation equations in terms of horizontally-averaged equations [*J Finnigan*, 2000; *M. R. Raupach and Shaw*, 1982]. This operation both simplifies and complicates the problem. The operation of spatial and temporal averaging leads to the introduction of new terms, called dispersive fluxes. But they also lead to a set of equations that are not point specific.

In their classic analysis of spatial averaging, Rapauch and Shaw [*M. R. Raupach and Shaw*, 1982]discuss two averaging schemes. Scheme I averages the instantaneous flow horizontally across a large plane to eliminate variation from plant elements or large scale eddies:

$$u(z,t) = <u> + u''(z,t)$$

where one denotes spatial averaging by double brackets <> and departure from the horizontal mean by ".

Scheme II time averages three dimensional flow, and horizontally averages the flow across a plane large enough to eliminate effects of plant elements:

$$\overline{u}(z,t) = <\overline{u}> + \overline{u}''(z,t)$$

Under scheme two one develops a time averaged version of the Navier-Stokes equation (du/dt) at a single point then is averaged across a plane large enough to eliminate effects of plant structure variation. When one assumes steady-state conditions, a horizontally homogeneous canopy, schems I and II should converge.   Raupach and Shaw show that their first averaging scheme yields:

$$\frac{1}{2}\langle u_i u_i \rangle = \frac{1}{2}\langle u_i \rangle \langle u_i \rangle + \frac{1}{2}\langle u_i'' u_i'' \rangle$$

Their averaging scheme II yields

$$\frac{1}{2}\left\langle u_i u_i \right\rangle = \frac{1}{2}\left\langle \overline{u_i}\,\overline{u_i} \right\rangle + \frac{1}{2}\left\langle \overline{u_i ' u_i '} \right\rangle =$$

$$\frac{1}{2}\left\langle \overline{u_i} \right\rangle \left\langle \overline{u_i} \right\rangle + \frac{1}{2}\left\langle \overline{u_i}"\overline{u_i}" \right\rangle + \frac{1}{2}\left\langle \overline{u_i ' u_i '} \right\rangle$$

The term, $\left\langle \overline{u_i}"\overline{u_i}" \right\rangle$, is denoted as a dispersive flux. A covariance occurs between two variables when there is spatial correlation among them.

Ayotte et al. (1999) [*Ayotte et al.*, 1999] articulate the spatial averaging rules as:

$$\mathrm{A}_j = \overline{A_j} + A_j '$$
$$\overline{A_j} = \left\langle \overline{A_j} \right\rangle + \overline{A_j}"$$
$$\mathrm{A}_j = \left\langle \overline{A_j} \right\rangle + \overline{A_j}" + A_j '$$

$$\left\langle \overline{A_j}" \right\rangle = 0$$

They also stress some rules relating to continuity, which holds for instantaneous, spatially averaged, time and space averaged and spatial fluctuations:

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial \left\langle u_i \right\rangle}{\partial x_i} = \frac{\partial \left\langle \overline{u_i} \right\rangle}{\partial x_i} = \frac{\partial \overline{u_i}"}{\partial x_i} = 0$$

The horizontal average for u is defined across a region in or above the canopy as:

$$< u >= \frac{1}{A}\int\int_R u(x,y,z)dxdy$$

Derivative operations are quite complex. Let’s consider pressure fields across an series of elements in the wind.

$$\left\langle \frac{\partial c}{\partial x} \right\rangle = \frac{\partial \left\langle c \right\rangle}{\partial x}$$

$$\left\langle \frac{\partial c"}{\partial x} \right\rangle \neq 0$$

At a particular point in the wind field downwind from an obstruction, $\frac{\partial \overline{p}}{\partial x} = \frac{\partial \overline{p}''}{\partial x} > 0$. We can also show that $\left\langle \frac{\partial p''}{\partial x} \right\rangle > 0$, but $\frac{\partial \left\langle \overline{p}'' \right\rangle}{\partial x} = 0$.

The spatial averaged equation for horizontal velocity is:

$$\frac{\partial \left\langle \overline{u} \right\rangle}{\partial t} = 0 = -\frac{\partial \left\langle \overline{w'u'} \right\rangle}{\partial z} - \left(\left\langle \frac{\partial \overline{p}}{\partial x} \right\rangle + \left\langle \frac{\partial \overline{p}''}{\partial x} \right\rangle \right) + \nu \left(\frac{\partial^2 \left\langle \overline{u} \right\rangle}{\partial z \partial z} + \left\langle \frac{\partial^2 \overline{u}''}{\partial z \partial z} \right\rangle \right) - \left\langle \overline{w} \frac{\partial \overline{u}}{\partial z} \right\rangle$$

The spatial averaged equation for turbulent kinetic energy is:

$$\frac{\partial \overline{e}}{\partial t} = 0 = -\left\langle \overline{w'u'} \right\rangle \frac{\partial \left\langle \overline{u} \right\rangle}{\partial z} + \frac{g \left\langle \overline{w'\theta'} \right\rangle}{\left\langle \overline{\theta} \right\rangle} - \frac{\partial \left\langle \overline{w'e'} \right\rangle}{\partial z} - \frac{\partial \left\langle \overline{p'w'} \right\rangle}{\rho \partial z} - \left\langle (\overline{u_i ' u_j '})'' \frac{\partial \overline{u}''}{\partial x_j} \right\rangle - \left\langle \varepsilon \right\rangle$$

The spatial averaged equation for momentum transfer is:

$$\frac{\partial \left\langle \overline{u'w'} \right\rangle}{\partial t} = 0 = -\left\langle \overline{w'u'} \right\rangle \frac{\partial \left\langle \overline{u} \right\rangle}{\partial z} - \frac{\partial \left\langle \overline{w'w'u'} \right\rangle}{\partial x} + \overline{\left\langle p' \left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right) \right\rangle} - \frac{\partial \left\langle \overline{u'p'} \right\rangle}{\partial z}$$

For simplicity we are assuming that the dispersion and wake productions terms are negligible in the momentum budget and are ignoring waving effects in the tke budget. Full derivations are provided in Finnigan (2000) and Ayotte et al. (1999).

Second order Approximations

An effective eddy exchange coefficient is typically applied to close third order terms [*M.R. Raupach*, 1988]. It follows the form:

$$\overline{w' u_i ' u_i '} = -K \frac{d \overline{u_i ' u_i '}}{dz}$$

This allows the third order moment to be computed in terms of the second order terms:

$$\overline{u_i'u_j'u_k'} = -q\lambda_1 \left(\frac{\partial \left\langle \overline{u_i'u_j'} \right\rangle}{\partial x_k} + \frac{\partial \left\langle \overline{u_j'u_k'} \right\rangle}{\partial x_i} + \frac{\partial \left\langle \overline{u_k'u_i'} \right\rangle}{\partial x_j} \right)$$

q is the characteristic velocity

$$q = \sqrt{\overline{u_i'u_i'}}$$

and lambda is a coefficient.

Pressure gradients are parameterised as a function of a drag coefficient, the leaf area density and the wind velocity squared [*Katul and Albertson*, 1998; *N R Wilson and Shaw*, 1977].

$$\left\langle \frac{\partial \overline{p}''}{\partial x_i} \right\rangle = C_d a(z) \left\langle \overline{u} \right\rangle^2$$

Viscous drag is negligible in the momentum equation compared to **form drag** forces

$$\nu \left\langle \frac{\partial^2 \overline{u_i}''}{\partial x_j \partial x_i} \right\rangle = 0$$

For turbulence we have a system of 5 equations and 5 unknowns. The resulting equations with second order closure assumptions on the basis of steady state conditions and horizontal homogeneity (Wilson and Shaw, 1977; Meyers and Paw U, 1987; Wilson, 1988; Katul and Albertson, 1998) are:

Momentum transfer

$$0 = \frac{d \left\langle \overline{w'u'} \right\rangle}{dz} - C_d a(z) \left\langle \overline{u} \right\rangle^2$$

Reynolds' Stress budget

$$0 = -\left\langle \overline{w'w'} \right\rangle \frac{\partial \left\langle \overline{u} \right\rangle}{\partial z} + 2 \frac{d}{dz} \left(\frac{q\lambda_1 d \left\langle \overline{w'u'} \right\rangle}{dz} \right) - \frac{q \left\langle \overline{w'u'} \right\rangle}{3\lambda_2} + Cq^2 \frac{d \left\langle \overline{u} \right\rangle}{dz}$$

Variance Budget for longitudinal wind velocity

$$0 = -2\left\langle \overline{w'u'} \right\rangle \frac{\partial \left\langle \overline{u} \right\rangle}{\partial z} + \frac{d}{dz}\left(\frac{q\lambda_1 d \left\langle \overline{u'u'} \right\rangle}{dz} \right) + 2C_d a \left\langle \overline{u} \right\rangle^3 - \frac{q}{3\lambda_2}\left(\left\langle \overline{u'u'} \right\rangle - \frac{q^2}{3} \right) - \frac{2q^3}{3\lambda_3}$$

Variance Budget for Lateral wind velocity

$$0 = \frac{d}{dz}\left(\frac{q\lambda_1 d \left\langle \overline{v'v'} \right\rangle}{dz} \right) - \frac{q}{3\lambda_2}\left(\left\langle \overline{v'v'} \right\rangle - \frac{q^2}{3} \right) - \frac{2q^3}{3\lambda_3}$$

Variance Budget for vertical velocity

$$0 = 3\frac{d}{dz}\left(\frac{q\lambda_1 d \left\langle \overline{w'w'} \right\rangle}{dz} \right) - \frac{q}{3\lambda_2}\left(\left\langle \overline{w'w'} \right\rangle - \frac{q^2}{3} \right) - \frac{2q^3}{3\lambda_3}$$

$$\lambda_1 = a_1 kz$$
$$\lambda_2 = a_2 kz$$
$$\lambda_3 = a_3 kz$$

$$\left| \frac{dl}{dz} \right| \le k$$

$l=kz$

The molecular destruction rate of turbulent kinetic energy per unit mass is

$$\varepsilon = \frac{(V)^3}{l} = \nu \overline{\left(\frac{\partial u_i}{\partial x_k}\right)^2}$$

and is parameterised in terms of a velocity scale cubed and a length scale. Near the surface the velocity scale is proportional to u* and l is close to z.

$$\varepsilon = \frac{u_*^3}{z}$$

Viscous dissipation is linked to isotropic viscous dissipation (as occurs in the free atmosphere) and work against pressure and form drag of plant elements.

Dispersion and wake production terms arise in the tke and momentum budgets. They are important in the tke budget, but are negligible in the momemtum budget.

Third Order Closure

The budget equations for the second order moments, unfortunately, include additional unknowns of the third order (such as $\overline{w'w'u'}, \overline{w'w'c'}$). At the third order, Meyers and Paw U (1987) [*Meyers and Paw U*, 1987]define and solve 22 equations. The equations they used are listed below and are separated for the scalar and the wind field. The logic of attaining closure at orders two or three is an assumption that errors introduced at higher orders will have a minimal effect on the estimate of the flux and concentration field. Deardorff [*J.W. Deardorff*, 1972] perceptively criticizes the use of 'effective' exchange coefficients to close budget equations of higher order moments because of an ultimate reliance on down-gradient diffusion. Deardorff argues that 'effective' exchange coefficients are inadequate for near-field flows—which occur in the vicinity of sources and sinks. This is because any turbulent diffusivity, K, in the vicinity of a source or sink is linearly related to the time period that fluid parcels have travelled. Only after a long travel distance is the time independent, "far-field" limit of K reached. The dispersion of a scalar released by sources at different distances upwind from an observer (as in a plant canopy) cannot be described by a single effective diffusivity (Wilson, 1989), as is attempted in higher order closure schemes. Other criticisms of higher order closure models revolve around the use of certain laboratory-based model parameters and parameterization schemes in the natural environment (Wyngaard, 1988). Yet despite the criticisms listed above, Eulerian higher order closure models have successfully simulated temperature and wind speed profiles and fluxes of momentum, heat and moisture within and above crop canopies (Meyers and Paw U, 1986, 1987; Naot and Maherer, 1989). However, in a recent analysis, Katul and Albertson (1998) conclude that there is no clear advantage to close the wind and turbulence equation beyond the second order.

A typical third order moments assessed with a gradient-diffusion approximation is:

$$\overline{w'^3} = -K_{www} \frac{\partial \overline{w'w'}}{\partial z}$$

The effective exchange coefficient is defined from:

$$K_{www} = b_3 \frac{k}{\varepsilon} \overline{w'w'}$$

Triple moments for scalars are typically closed with the Quasi Guassian approximation (e.g. Meyers, 1985):

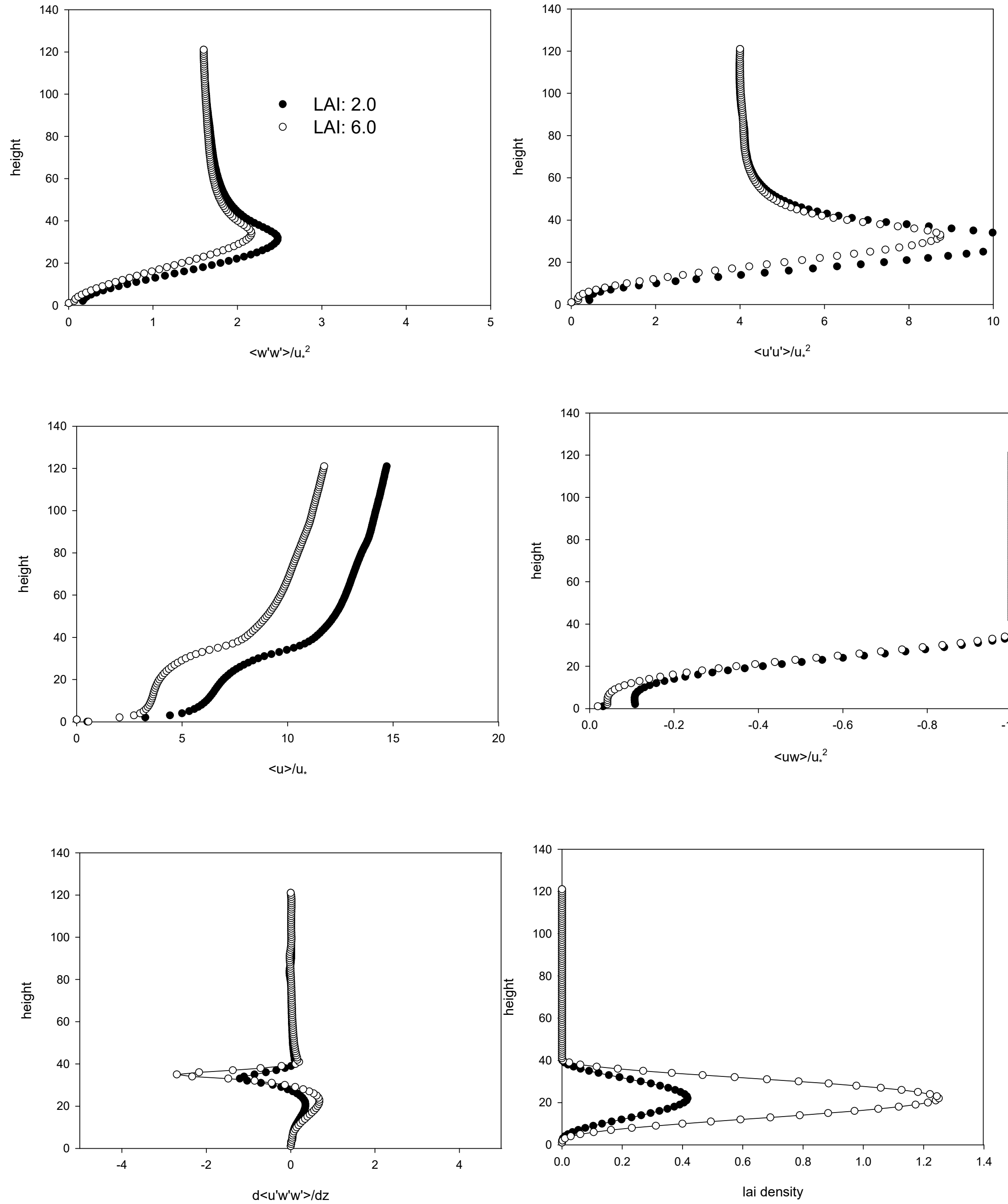
$$\overline{a'b'c'd'} = \overline{a'b'}\,\overline{c'd'} + \overline{a'c'}\,\overline{b'd'} + \overline{a'd'}\,\overline{b'c'}$$

A typical budget for a third order moment is:

$$\frac{\partial \overline{a'a'b'}}{\partial t} = 0 = -\overline{w'a'a'}\frac{\partial \bar{b}}{\partial z} - 2\overline{w'a'b'}\frac{\partial \bar{a}}{\partial z} - \overline{w'b'}\frac{\partial \overline{a'a'}}{\partial z} - 2\overline{w'a'}\frac{\partial \overline{a'b'}}{\partial z} - \varepsilon_{aab}$$

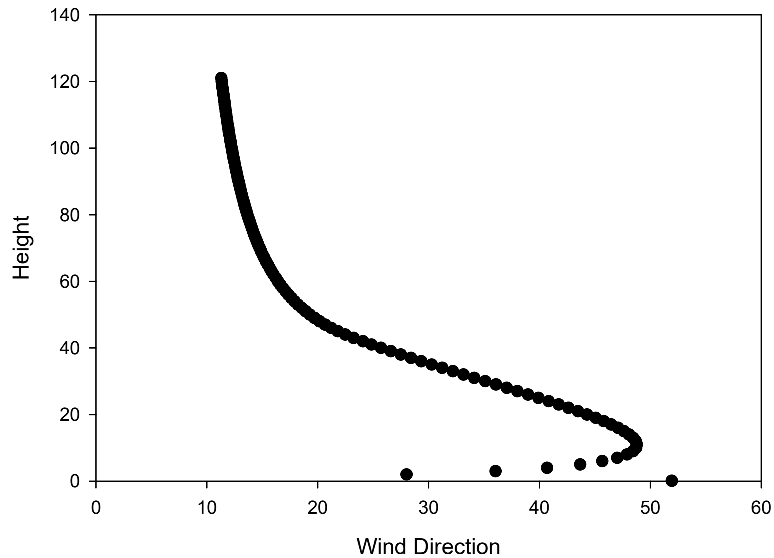
Simulations

With the higher order closure model of Meyers and Paw U we can explore the effects of leaf area index and its vertical distribution the wind and turbulence within and above a canopy. Below are simulations for a sparse (LAI=2) and dense (LAI=6) canopy. Note shifts in wind profiles, the change in shear, the generation of turbulence (w'w' and u'u') where leaf area is dense and the significant transport terms.



LAI: 2.0
LAI: 6.0
height
$\langle w'w' \rangle / u_*^2$
$\langle u'u' \rangle / u_*^2$
$\langle u \rangle / u_*$
$\langle uw \rangle / u_*^2$
d$\langle u'w'w' \rangle$/dz
lai density

Lai = 4



APPENDICES

1. Boussinesq Approximation

Boussinesq approximation for density fluctuations.
Flow is incompressible, so:

$$\frac{\partial u_j'}{\partial x_j} = 0 = \frac{\partial u_j}{\partial x_j} = \frac{\partial \overline{u_j}}{\partial x_j}$$

$$\rho(t, x, y, z) = const$$
$$\rho(T) \neq const$$

Leading to
$$\frac{\rho'}{\rho_r} << 1 ;\ \frac{T'}{T_r} << 1 ;\ \frac{P'}{P_r} << 1$$

but density fluctuations are coupled with the acceleration of gravity in the Navier Stokes Equation to yield significant forces.

1. Reference State

$$P_r = \rho_r R T_r$$

$$\frac{dP_r}{dz} = -\rho_r g$$

$$\frac{dT_r}{dz} = -\frac{\rho_r}{C_p}$$

$$\ln P_r = \ln \rho_r + \ln R + \ln T_r$$

$$\frac{dP_r}{P_r} = \frac{d\rho_r}{\rho_r} + \frac{dT_r}{T_r} \approx 0$$

$$\frac{d\rho_r}{\rho_r} \approx -\frac{dT_r}{T_r}$$

so in essence fluctuations in density can be approximated with temperature fluctuations

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