

Lecture 10 Lagrangian Models

ESPM 228, Advanced Topics on Micrometeorology and Biometeorology

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1. Lagrangian Models

- a. Statistical Model of Turbulent Diffusion, Taylor Theory
- b. Langevin Equation
- c. The role of Heterogeneous turbulence
- d. Localized Near Field Theory

2. Large Eddy Simulation

Lagrangian Models

The Lagrangian coordinate framework considers the position of a particle (x, y, z) at time t relative to its initial position. In the Lagrangian frame, a concentration field and its turbulent flux are defined by the statistics of an ensemble of dispersing marked fluid parcels and the strength and spatial distribution of sources and sinks [M. R. Raupach, 1989a; Sawford, 1985; J.D. Wilson, 1989]. This approach is valid as long as the molecular diffusion within a fluid parcel is negligible in comparison to its turbulent diffusion through the atmosphere. Solid reviews on Lagrangian diffusion [Rodean, 1996; Sawford, 1985; Thomson, 1987] and application into plant canopies [M. R. Raupach, 1987; M.R. Raupach, 1988; M. R. Raupach, 1989a; b; M. R. Raupach and Thom, 1981; J. D. Wilson and Sawford, 1996; J. D. Wilson *et al.*, 1981] can be found in a set of papers by such authors as Sawford, Raupach, Wilson and Thomson.

The concentration of a scalar can be defined at a particular location and after a given travel time using the principle of superposition.

From first principles, a concentration is simply defined as the number of fluid parcels (n) observed in a given volume (V) at a specified vector location (\mathbf{r}) and time (t):

$$C(\mathbf{r}, t) = \frac{n(\mathbf{r}, t)}{V}$$

To evaluate the concentration of material in time and space we introduce the concept of a joint conditional probability density function, $P(\mathbf{r}, t | \mathbf{r}_0, t_0)$. It defines the probability that a

fluid parcel released from a point in space (z_0) and time (t_0) will be observed at another location and time (z, t). For the case of vertical diffusion, the concentration observed at point z and time t is:

$$c(z, t) = \int_0^t \int_0^z P(z, t | z_0, t_0) S(z_0, t_0) dz_0 dt_0$$

The conditional probability of a fluid parcel, labelled X_n , equals one if it is released from a reference location (z_0) at an initial time (t_0) and arrives at a volume centered on location z at time t ; otherwise X_n is zero.

The probability density function for the diffusion of fluid parcels depends only on the properties of the turbulent wind field, which must be prescribed. The joint probability density function, $P(z, t | z_0, t_0)$, is difficult to define analytically in such conditions. Yet, $P(z, t | z_0, t_0)$ can be specified using a Markovian "random-walk" approach. This method computes the trajectory of a large number of fluid parcels. Methods for determining fluid parcel movement are derived from the Langevin equation and are discussed by [M.R. Raupach, 1988; Rodean, 1996].

Let's, consider the simplest case, the statistical properties of one dimensional diffusion, as identified by Taylor [Taylor, 1935]. The displacement of fluid parcels can be computed as a function of time as a product of the vertical velocity and time step :

$$dx = u_L \cdot dt$$

where u_L is the Lagrangian velocity ($m s^{-1}$) and dt is differential time increment.

Incremental changes in vertical velocity were computed with the **Langevin equation**, a stochastic differential equation that is weighted by a deterministic forcing (which is a function of the fluid parcel's previous velocity, a memory term) and a random forcing term [Thomson, 1987]:

$$\frac{du}{dt} = -a_1 u + b \xi(t)$$

The term ξ is a random forcing function.

The concept of the Langevin equation evolved from the study of Brownian motion by Einstein, back in 1905. Other work on Brownian motion is attributed to Markov, the famous Russian Mathematician.

The white noise term has several distinct properties. It possesses a uniform spectral density. The term is discontinuous causing sequential values to be decoupled from and unrelated to the previous value. Consequently one can define a continuous integral,

$W(T) = \int_0^T \xi(t) dt$, but not a differential, as $\frac{dW(t)}{dt}$ is undefined. On the other hand we can define an incremental Wiener process: $dW(t) = \xi(t) dt$

For the incremental Wiener process (Arnold, 1979) we ascribe to several rules:

$$\langle dW(t) \rangle = 0$$

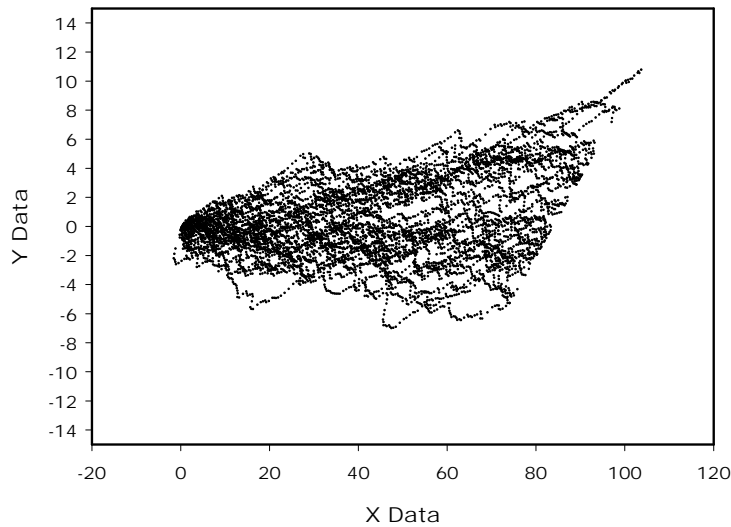
$$\langle |dW(t)|^2 \rangle = dt$$

Its mean is defined as:

$$\langle W(t) \rangle = w_0$$

and its variance is:

$$\langle (W(t) - w_0)^2 \rangle = t - t_0$$



The terms $a(z, t, w_L)$ and $b(z, t, w_L)$ are derived from the budget equation for the Eulerian probability density function of w_L , the Fokker-Planck equation (Thomson, 1987; Rodean,

1996). The Fokker-Planck equation is a partial differential equation that describes the behavior of the conditional joint probability density function, P.

$$\frac{\partial P(x,u,t)}{\partial t} + u \frac{\partial P(x,u,t | x_0, u_0, t_0)}{\partial x} = - \frac{\partial (a_0 - a_1 u) P(x,u,t | x_0, u_0, t_0)}{\partial u} + \frac{1}{2} \frac{\partial^2 [b^2 P(x,u,t | x_0, u_0, t_0)]}{\partial u^2}$$

The coefficients a0, a1 and b are equivalent to those produced in the Langevin equation. Its Eulerian equivalent of the Lagrangian Langevin equation:

A visual example for a simple one-dimensional case is shown below:

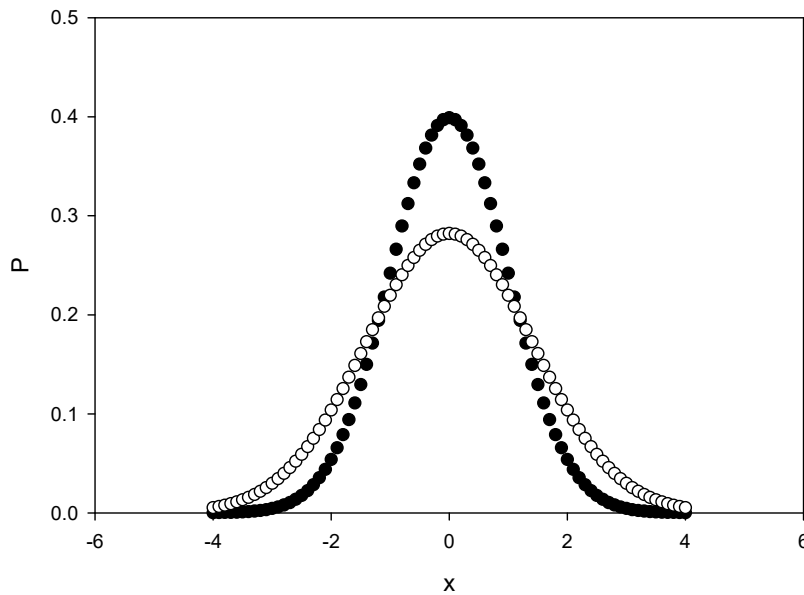


Figure 1 Probability distribution spreading. More variance.

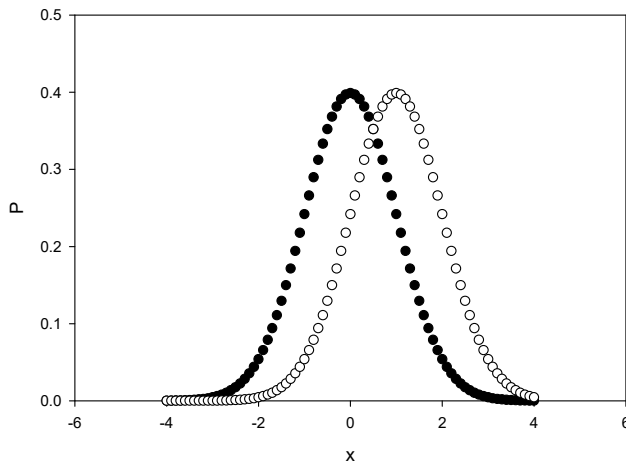


Figure 2 Probability distribution translation, or ‘aging’

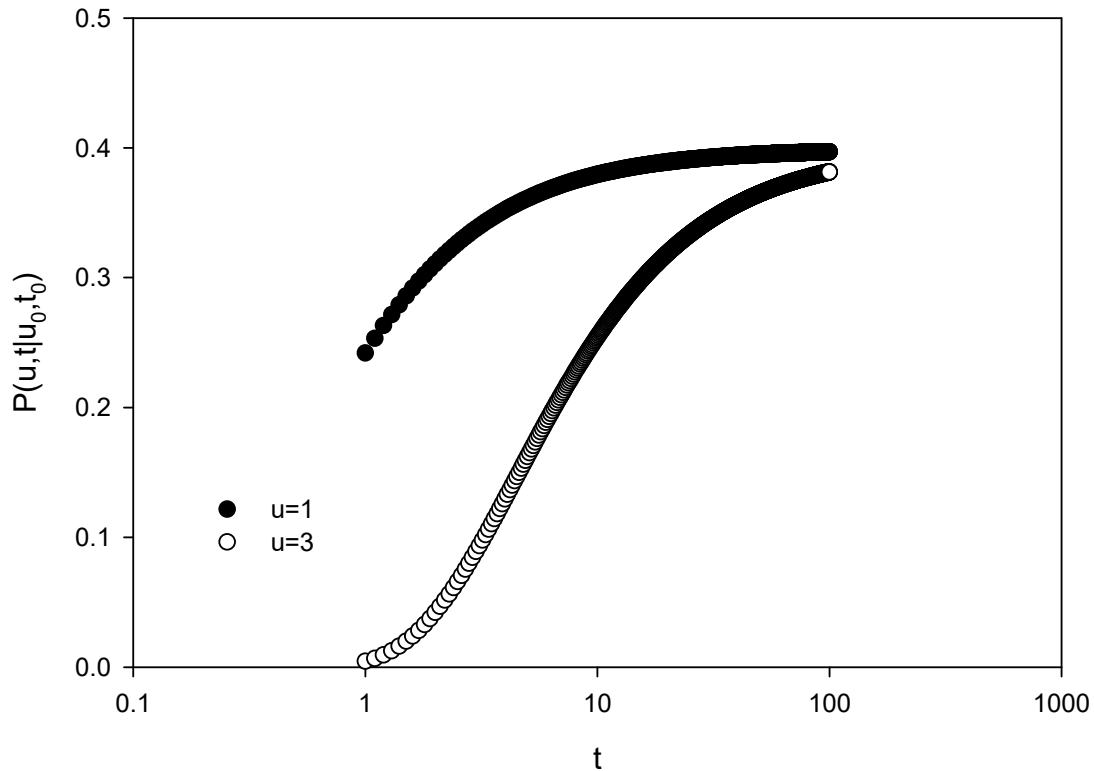
At an instant in time and a given location in space, the Lagrangian and Eulerian velocities are equal.

From the Fokker-Planck equation one can deduce that the shape of a joint pdf will vary due to ‘advection’ and ‘diffusion.’ The Fokker-Planck equation leads to a form reminiscent of the famous Gaussian diffusion equation when a_0 and a_1 are zero and b is one.

$$\frac{\partial P(u, t | u_0, t_0)}{\partial t} = \frac{1}{2} \frac{\partial^2 [P(u, t | u_0, t_0)]}{\partial u^2} \text{ when one applies the initial condition}$$

$$P(u, t | u_0, t_0) = \delta(u - u_0)$$

$$P(u, t | u_0, t_0) = \frac{1}{\sqrt{2\pi(t-t_0)}} \exp\left(-\frac{(u-u_0)^2}{2(t-t_0)}\right)$$



Typically, the Gaussian plume diffusion equation represents how the concentration of a scalar varies as the plume grows, knowing the source strength Q , (mass length⁻²):

$$C = \frac{Q}{\sqrt{2\pi\sigma_z^2}} \exp\left(-\frac{(z-z_s)^2}{2\sigma_z^2}\right)$$

Considering a continuous source, then one solves

$$\frac{\partial c}{\partial t} = \frac{q}{\sqrt{2\pi\sigma_z^2}} \exp\left(-\frac{(z-z_s)^2}{2\sigma_z^2}\right)$$

where q is a continuous source with units of mass length⁻² time⁻¹.

These probabilities are a function of the initial velocity and times.

Use of the Fokker-Planck equation (to evaluate the Langevin equation) was recommended in the late 1950s by Obukhov, but this suggestion was met with resistance. The popularity of the Langevin equation did not rise until the late 1970s and 1980s when it was used by Hanna and Gifford at the Atmospheric Turbulence and Diffusion

Laboratory. It was soon applied in canopies by Wilson, Legg and Raupach, but with poor success due to the non-uniform turbulent field in the canopy. Theoretical breakthroughs came in 1987 with work by Thomson and his formal derivation of the Langevin equation, based on the Fokker-Planck equation. As Rodean and Sawford explain, Lagrangian statistics will be used to solve for b and Eulerian statistics will be used to solve for a_0 and a_1 .

Turbulent Diffusion is a Homogeneous Field

To gain a better understanding of the nuances of Lagrangian turbulent diffusion it is instructive to first consider diffusion under ideal conditions, where turbulent statistics are distributed in a Gaussian manner and the turbulent field is invariant in space (it is homogeneous).

The classic description of turbulent motion in homogeneous turbulence is attributed to Taylor (1921). Taylor was the first to treat particle movement for turbulent flow. In contrast his predecessors (Markov, Langevin) dealt with Brownian motion. Independent of these early studies, Taylor appreciated the impact of random stochastic motions upon the mean turbulent regime.

The derivation of Taylor's analysis is instructive as it helps us understand several fundamental features of Lagrangian diffusion, e.g. near and far field diffusion, which have helped us understand counter gradient transfer in turbulent flow. For simplicity we will only consider vertical diffusion since it is most applicable to our examination of diffusion in and above a plant canopy. From first principles, the position of a fluid parcel is defined by integrating its velocity over a given time interval, which leads to a derivation of Taylor's Diffusion Law. The time rate of change of the spread of a plume, in terms of its variance:

$$\frac{dZ^2}{dt} = 2Z \frac{dZ}{dt}$$

$$Z = \int_0^t W dt$$

$$\frac{dZ}{dt} = W$$

$$\frac{dZ^2}{dt} = 2ZW = 2 \int_0^t WW dt$$

$$WW = \sigma_w^2 r_L$$

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 \int_0^t r_L(\tau) d\tau$$

where we define the correlation coefficient as:

$$r_L = \frac{\langle W'(t)W'(t+\tau) \rangle}{\langle W'^2 \rangle}$$

Lagrangian time scale, a measure of the persistence of fluid motion

$$T_L = \int_0^\infty r_L(\tau) d\tau$$

Two very important and commonly cited limits can be determined from this analysis, the near and far field limits. It is crucial to defined these limits as many contemporary diffusion models (Raupach, 1987; Warland and Thurtell, 2000) use these limits in their derivations. These limits also affect the distinct properties of turbulent diffusion and concentrations fields in plant canopies, such as counter-gradient diffusion.

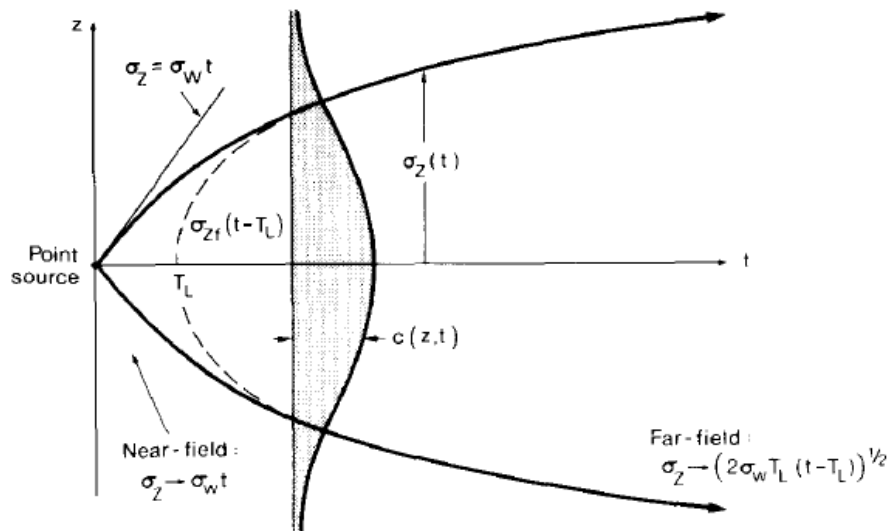


Figure 3 After Raupach 1989 [M. R. Raupach, 1989c]

Near Field

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 \int_0^t r_L(\tau) d\tau$$

Motions are well correlated with one another, showing persistence, so the correlation coefficient is one.

$$r_L = 1$$

$$\int_0^t r_L(\tau) d\tau = t$$

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 t$$

Separate Variables and Integrate with respect to t

$$\int d\sigma_z^2 = \int 2\sigma_w^2 t dt$$

$$\sigma_z^2 = \sigma_w^2 t^2$$

So we arrive at an equation that states that the standard deviation of the size of a diffusing plume is a function of the standard deviation in vertical velocity fluctuations and the travel time, when the travel time is less than the integral time scale:

$$\sigma_z \sim \sigma_w t, t \ll T_L$$

Far Field Diffusion

Far field diffusion refers to the case where travel time exceeds the turbulence time scale. Parcel motion in the far field is not correlated with initial conditions and can be assumed random. The integration of the autocorrelation function approaches the value of the Lagrangian turbulence

Far field limit is defined from the following:

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 \int_0^t r_L(\tau) d\tau$$

$$t \rightarrow \infty$$

$$\int_0^\infty r_L(\tau) d\tau = T_L$$

$$\int d\sigma_z^2 = \int 2\sigma_w^2 T_L dt$$

$$\sigma_z^2 = 2\sigma_w^2 T_L t$$

In this situation the plume dispersion is a function of the square root of travel time.

$$\sigma_z = \sigma_w \sqrt{2T_L t}$$

If we compare plume size and travel time we note that the near field plume is much tighter for short travel times.

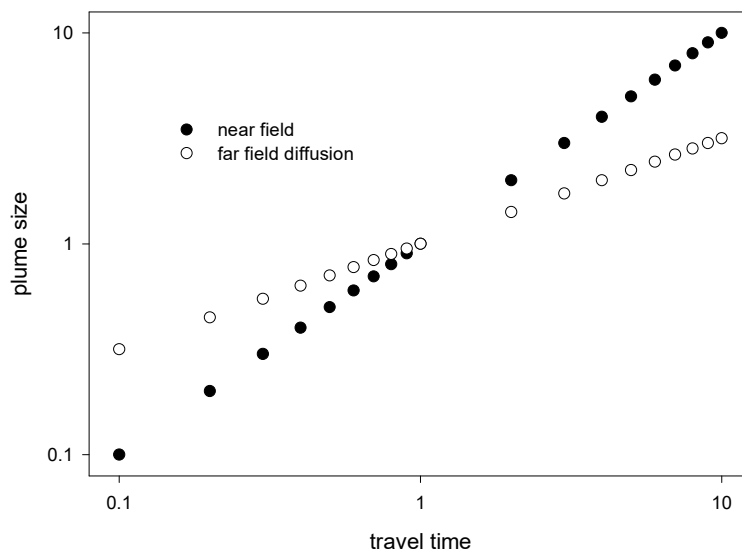


Figure 4 Visualization of plume size growth in the far and near field diffusion regimes.

$$z(t) = \int_0^t w(t') dt'$$

In homogeneous turbulence, the Langevin Equation can be solved analytically. For completeness we give the solution [Legg and Raupach, 1982]:

$$W(t) = W(0) \exp(-a_1 t) + b \exp(-a_1 t) \int_0^t \exp(a_1 s) \xi(s) ds$$

where $W(t)$ is considered to be a function of its initial value ($W(0)$) and a random term. The mean of the random term is zero., as is mean vertical velocity. The covariance between random increments at t and s is:

$$\langle \xi(s)\xi(t) \rangle = \delta(t-s)$$

An important measure for the determination of the spreading of motion (ie. the spreading of a cloud) is the variance of the vertical motion around its mean value. This variance can be derived by taking the mean value of the W' squared:

$$\overline{W'W(t)'} = \overline{W'W(0)'} \exp(-2\alpha t) + b^2 \int_0^t \exp(a_1(s-t)) \xi(s) ds$$

Evaluating the integral yields:

$$\overline{W'W(t)'} = \overline{W'W(0)'} \exp(-2a_1 t) + \frac{b^2}{2a_1} (1 - \exp(-2a_1 t))$$

The coefficients α and b can be derived by considering the covariance for w between time 0 and t :

$$\overline{w(t)'w(0)'} = \overline{w'w'(0)'} \exp(-a_1 t)$$

The coefficient α is a function of the time scale of the motion, since the correlation of w between

$\alpha = \frac{1}{T_L}$ time 0 and time t decreases as the time proceeds. It is defined as:

If the mean vertical velocity equals zero then:

$$b = \sigma_w \sqrt{2a_1} = \sigma_w \sqrt{2/T_L}$$

where σ_w is the Lagrangian velocity variance. For simplicity, it is assumed that the Lagrangian and Eulerian velocity variances are identical.

Leading to a workable form of the Langevin equation for Homogeneous turbulence

$$dw = -\frac{w}{T_L} dt + \sqrt{\frac{2\sigma_w^2}{T_L}} \xi(t) dt$$

The time step Δt must be much greater than T_a , the acceleration time scale; T_a is in order of $T_L Re^{-1/2}$ (Tennekes and Lumley, 1972). On the other hand, the period between time steps must be small in inhomogeneous turbulence to keep the fractional change in T_L small between time steps. Δt cannot exceed the Lagrangian time scale T_L without losing essential information about the motion (Sawford, 1984). Thus $T_a \ll \Delta t \ll T_L$.

Heterogeneous Gaussian Turbulence

Inside a plant canopy, turbulence is inhomogeneous and non-Gaussian (Wilson et al., 1982; Raupach, 1988; Baldocchi and Meyers, 1988a, 1988b); length, time and velocity scales of turbulence vary with height and the probability density functions of the three vector velocity components are skewed and kurtotic. The joint probability density function, $P()$, cannot be specified analytically in non-Gaussian, inhomogeneous turbulence. Over the past twenty years numerous investigators tried to develop algorithms for simulating particle movement in canopies with mixed success.

Lagrangian models based on the Langevin equation will fail where the vertical velocity variance increases with height, as occurs inside plant canopies. The vertical gradient in the vertical velocity variance imposes a downward drift on a Markovian random flight model. This net downward drift occurs because downward directed fluid parcels from above enter a lower region with a decreased vertical velocity scale and a reduced probability of leaving that region [J. D. Wilson and Sawford, 1996] (Raupach, 1988; Sawford, 1985; Wilson, 1989). Accumulation of matter near the surface would otherwise occur in nature if it were not for the intermittent gusts that disproportionately transfer matter and maintain continuity. Heuristic arguments have been used to develop approaches to remove the unrealistic accumulation of matter that would otherwise occur when using Markov sequence models in a field of inhomogeneous turbulence. One approach introduces an additional force term into the Langevin equation. This method yields a mean upward drift velocity in the solution of the differential equation (Wilson et al., 1981; Legg and Raupach, 1982). Another method by-passes the addition of a bias velocity by reflecting marked particles according to a probability calculated from the gradient in the vertical velocity variance (Leclerc et al., 1988).

A series of papers by David Thomson [Thomson, 1987] brought order to the problem of simulating Lagrangian diffusion in plant canopies. Thomson applied the Fokker-Planck equation to derive a similar, but statistically defensible relationship. In his classic paper, Thomson outlines several criteria that a random walk model must meet.

1. the well-mixed criterion must hold: if particles are initially well-mixed they will remain so in turbulent flow
2. Eulerian models derived from the Lagrangian model must be compatible with Eulerian equations that are derived from first principles.
3. the model will reduce to the diffusion equation when the Lagrangian time scale approaches zero

4. the forward and reverse formulations of dispersion are consistent
5. the small time behavior of the velocity distribution of particles must be correct.

Thomson (1987) [Thomson, 1987] wrote a classic paper in 1987 that resolved many of the problems associated with predicting turbulent diffusion. He started with a rigorous definition of the Langevin equation

$$dw = -a_1(z, t, w)dt + b(z, t, w)\xi(t)dt$$

The coefficients $a(z, t, w)$ and $b(z, t, w)$ are non-linear functions of w and are defined to account for inhomogeneous turbulence. The term $d\xi$ defines a Gaussian, white noise random forcing with a mean of zero and a variance of dt .

For the one-dimensional case, where velocity fluctuations and gradients occur only in the vertical direction, inhomogeneous turbulence, application of the Fokker-Planck equation to the Langevin equation yields:

$$dw_L = \left(-\frac{w_L}{T_L} + \frac{1}{2} \left[1 + \frac{w_L^2}{\sigma_w^2}\right] \frac{\partial \sigma_w^2}{\partial z}\right) dt + \sqrt{\frac{2\sigma_w^2}{T_L}} \xi(t) dt$$

(Wilson et al., 1983; Thomson, 1987). The middle term is a drift term and compensates for the impact of heterogeneous turbulence. *Ad hoc* assumptions are still applied. Most notable is the lower boundary condition that reflects particles.

Heterogeneous Gaussian and non-Gaussian turbulence

It is important to note that the derivations we have discussed so far are based on the assumption of Gaussian, random forcing. While one may define a non-Gaussian model, conceptually. In practice one is only able to approximate the criteria for a properly designed model (well-mixed criterion, a continuous Markov process). From a practical standpoint the assumption of Gaussian forcing may not be that wrong. We (Liukang Xu and I) have looked at this assumption with our turbulence data and found that the pdf of dT/dt is nearly Gaussian even though the pdf of T is skewed and kurtotic. This concept has been elaborated and formalized in by past work by Katul.

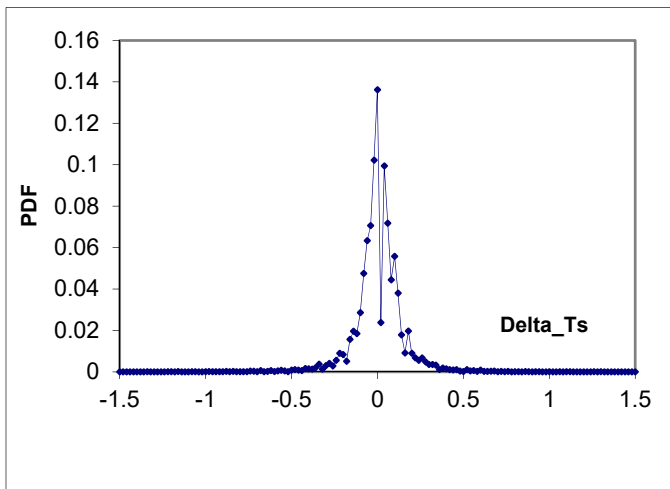


Figure 5 pdf of dT/dt for temperature over a grazed grassland.

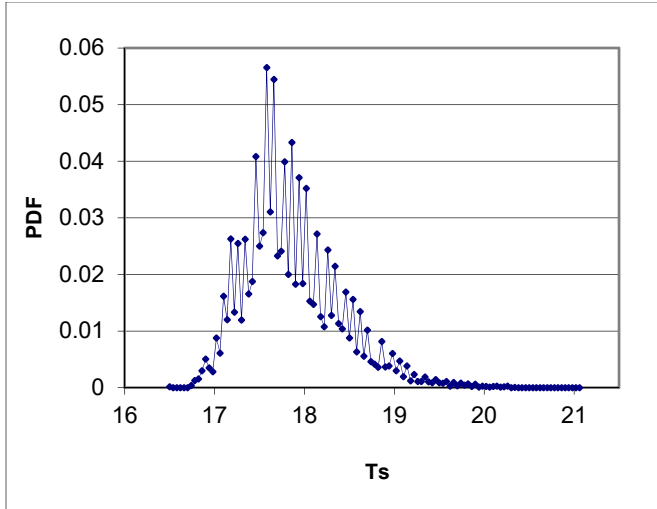


Figure 6 pdf of Temperature over a grazed grassland

From a numerical standpoint, Wilson, Sawford and others have commented that adding non-Gaussian increments is illposed and does not yield a suitable numerical solution.

Application

Before we can test the random walk models we address several issues. Of foremost importance we must choose the proper time step for parcel motion and we need to know how many fluid parcels to release.

Trajectories of fluid parcels were computed with a time step (Δt) equal to $0.025 T_L(h)$. This choice is based on sensitivity tests of the model. An equilibrium flux profile is commonly observed for mass, energy and momentum transfer over a surface in a fully-developed boundary layer. This is not reached until Δt is equal to or smaller than $0.05 T_L(h)$. The concentration profile, on the other hand, is less affected by the time step.

The number of particles is also crucial in generating numerically stable profiles with a random walk model. An optimal number of released parcels must be determined to economize on computer time, yet reduce the statistical noise associated with an inadequate number of particles. We found that 5000 parcels is sufficient to achieve a stable and repeatable concentration and flux profile. Smaller numbers of release parcels results in 'noisy' profiles and run-to-run variability.

The ultimate test of a random-walk model is whether or not it attains mass conservation. For the one-dimensional case, the canopy source. The amount of material released per unit area must equal the integral of concentration profile. In other words all the material released must be accounted for in the atmosphere.

In practice the source-sink function, S , is dependent on local concentration, c , and vice versa. Raupach (1988) devised a scheme that computes the interdependence between S and c . The concentration difference between an arbitrary level (c_i) and reference level (c_r) located above a plant canopy can be expressed as by summing the contributions of material diffusing to or from layers in the canopy (denoted by the subscript j):

$$c_i - c_r = \sum_{j=1}^N S_j(c_j) D_{i,j} \Delta z_j$$

The dispersion matrix ($D_{i,j}$) is solely a function of the turbulence statistics and can be computed by uniformly releasing fluid parcels from each canopy level.

$$D_{ij} = \frac{C_i - C_r}{S_j \Delta z_j}$$

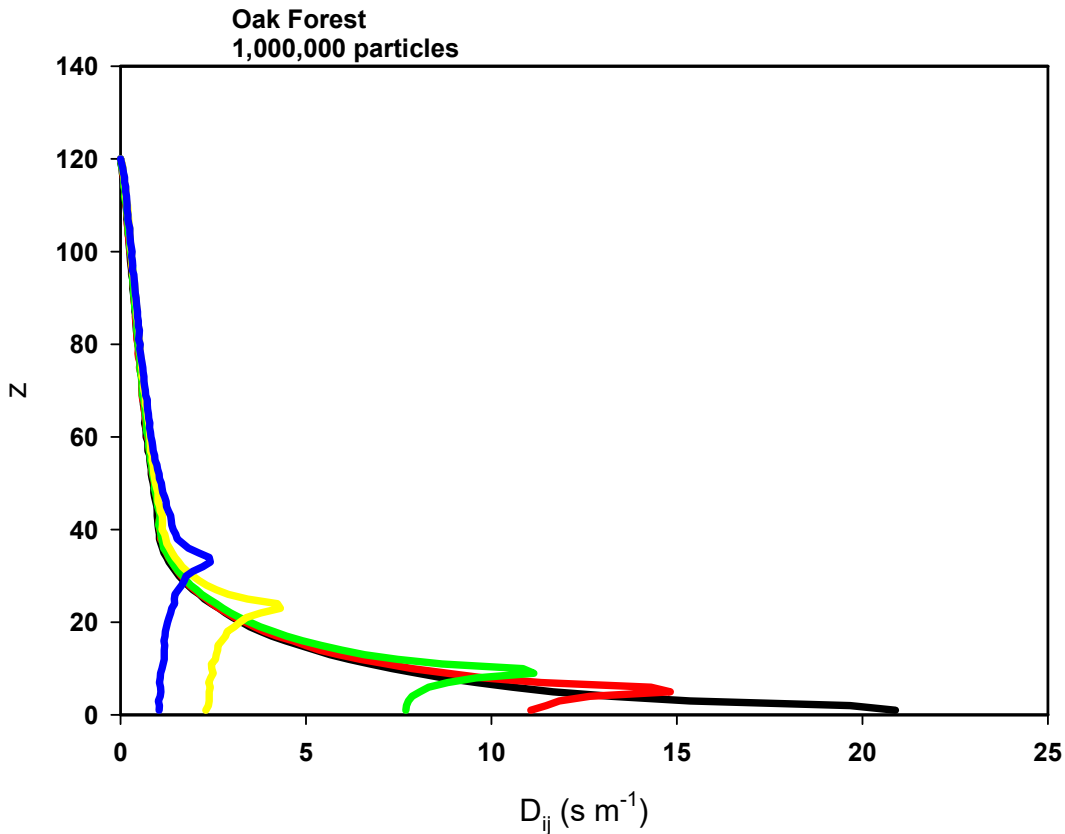


Figure 7 Dispersion matrix as a function of height and release height.

Note how the dispersion matrix varies with depth and the nose reflects the combined impacts of the local source strength and degree of local and distant turbulent mixing. Dispersion matrices are near zero for well-mixed flows. The advantage of using a numerical model as opposed to an analytical one is that one is able to incorporate as

much information as necessary on local source sink distributions and how turbulence is affected by stability and the distribution of foliage.

Our calculations have shown that we need to compute D for different stability classes.

Analytical Near Field/Far Field Theory of Raupach (1989)

The concentration of a traces gas in the atmosphere is the sum of the near and far field components:

$$C(z) = C_n(z) + C_f(z)$$

The far field concentration field can be solved from the K theory diffusion equation

$$\frac{\partial}{\partial z} (K(z) \frac{\partial C_f(z)}{\partial z}) = -S_c(z)$$

so we can also define

$$F(z) = -K(z) \frac{\partial C_f}{\partial z} = F_c + F_s$$

$$F_c(z) = \int_0^z S(z') dz'$$

$$K(z) = \sigma_w^2(z) T_L(z)$$

Raupach defines an algorithm for C_n

$$C_n(z) = \int_0^\infty \frac{S(z')}{\sigma_w'} \left\{ k_n \frac{z - z'}{\sigma_w(z') T_L(z')} + k_n \frac{z + z'}{\sigma_w(z') T_L(z')} \right\} dz'$$

$$k_n(x) \approx -0.3984 \ln(1 - \exp(-|x|)) - 0.1562 \exp(-|x|)$$

Scaling relations, we learned about last semester, can be applied here for near neutral conditions

$$\frac{\sigma_w}{u_*} = \begin{cases} a_1, & z > h \\ a_0 + (a_1 - a_0) \frac{z}{h}; & z \leq h \end{cases}$$

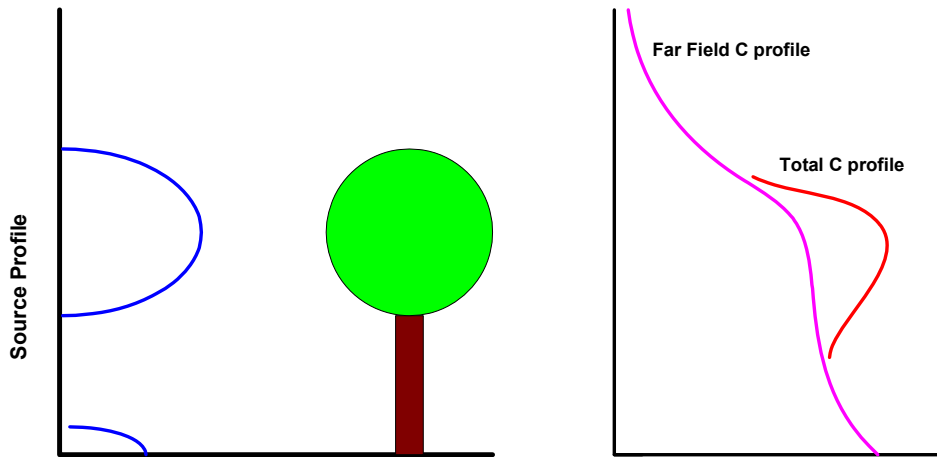
$$\frac{T_L u_*}{h} = \begin{cases} C_o; & \frac{k}{a_1^2 h} (z-d) \leq C_o \\ \frac{k}{a_1^2 h} (z-d); & \frac{k}{a_1^2 h} (z-d) \geq C_o \end{cases}$$

Typical values for a_0 , a_1 and C_0 are 0.25, 1.25 and 0.3.

$$C_f(z) - C_f(z_r) = \int_z^{z_r} \frac{1}{\sigma_w^2(z') T_L(z')} \left[\int_0^{z'} S(z'') dz'' + F_s \right] dz'$$

$$C_f(z) - C_f(z_r) = \int_z^{z_r} \frac{F(z)}{K_f} dz$$

$$C_f(z_r) = C(z_r) - C_n(z_r)$$



For the far field case, the diffusion of a random particle is independent of its travel time. This limit occurs when the travel time exceeds the Lagrangian time scale, T_L . The probability distribution of the spread of an ensemble of particles can be estimated as a function of the square root of the travel time. This behavior is denoted as far field diffusion.

Warland and Thurtell [Warland and Thurtell, 2000] (2000) revisited this problem and derived an algorithm for the dispersion matrix. One criticism of LNF theory identified by Warland and Thurtell is that it assumes homogeneous dispersion within fields of heterogeneous turbulence. Using near and far field limits and the Gaussian plume model discussed above and the conservation of mass and the identity

$$\frac{\partial}{\partial z} \left(\frac{\partial c}{\partial t} \right) = \frac{\partial}{\partial t} \left(\frac{\partial c}{\partial z} \right)$$

They produce:

$$\frac{\partial c}{\partial z} = \int_0^{\infty} \frac{-q}{\sqrt{2\pi\sigma_z^2}} \frac{(z-z_s)}{\sigma_z^2} \exp\left(\frac{-(z-z_s)^2}{2\sigma_z^2}\right)$$

Integrating this equation for the near and far field limits of plume growth they produce:

Near field case, z/L_L goes to zero

$$\frac{\partial c}{\partial z} \Big|_{nf} = \frac{-q}{\sqrt{2\pi}\sigma_w |z-z_s|}$$

Far field case, z/L_L goes to infinity, where L_L is the Lagrangian length scale.

$$\frac{\partial c}{\partial z} \Big|_{ff} = \frac{-q}{2\sigma_w L_L}$$

Next Dispersion matrices are defined on the basis of the general relation:

$$\frac{\partial c}{\partial z} = Dq$$

$$D_{nf} = \begin{cases} \frac{-1}{\sqrt{2\pi}\sigma_w(z-z_s)}; z > z_s \\ \frac{1}{\sqrt{2\pi}\sigma_w(z_s-z)}; z < z_s \end{cases}$$

$$D_{ff} = \begin{cases} \frac{-1}{2\sigma_w L_L}; z > z_s \\ \frac{1}{2\sigma_w L_L}; z < z_s \end{cases}$$

Next they define a continuous dispersion matrix

$$D = \left\{ \begin{array}{l} \frac{-1}{2\sigma_w L_L [1 - \exp(-\sqrt{\frac{\pi}{2}} (\frac{z_i - z_s}{L_L}))]} \quad \frac{1}{2\sigma_w L_L [1 - \exp(-\sqrt{\frac{\pi}{2}} (\frac{z_i + z_s}{L_L}))]} ; z_i > z_s \\ \frac{-1}{2\sigma_w L_L [1 - \exp(-\sqrt{\frac{\pi}{2}} (\frac{z_s + z_i}{L_L}))]} ; z_i = z_s \\ \frac{1}{2\sigma_w L_L [1 - \exp(-\sqrt{\frac{\pi}{2}} (\frac{z_s - z_i}{L_L}))]} \quad \frac{1}{2\sigma_w L_L [1 - \exp(-\sqrt{\frac{\pi}{2}} (\frac{z_s + z_i}{L_L}))]} ; z_i < z_s \end{array} \right.$$

The authors take this theory farther in their paper and define numerical relations in finite difference form, but the work is basically an extension of the previous equation, but it contains a reflecting boundary condition at the ground.

Next we show Warland and Thurtells test of their model and a comparison with LNF theory. They conclude their model is in closer agreement with measurements inside the canopy because they account of better treatment of near field turbulent diffusion.

THE RELATIONSHIP BETWEEN A DISTRIBUTED SOURCE AND CONCENTRATION PROFILE 467

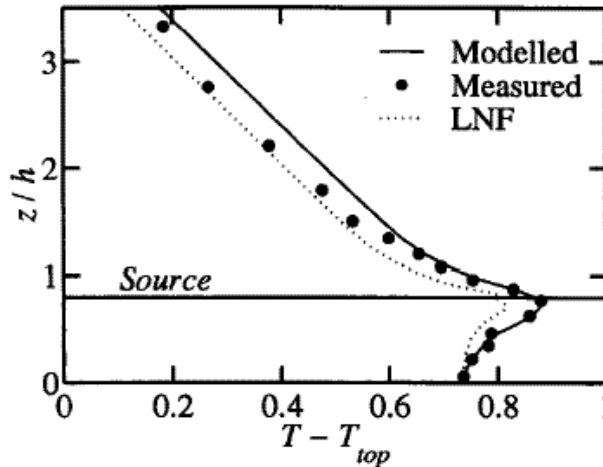


Figure 6. Comparison of CRL86's wind-tunnel data and the present model using Eulerian scales in place of the Lagrangian. Results from localized near-field theory also shown. Data are shown as the difference from the highest measured point T_{top} at $z/h \approx 6.3$.

Figure 8 From Warland and Thurtell (2000).

A comparison of source strength profiles is shown next

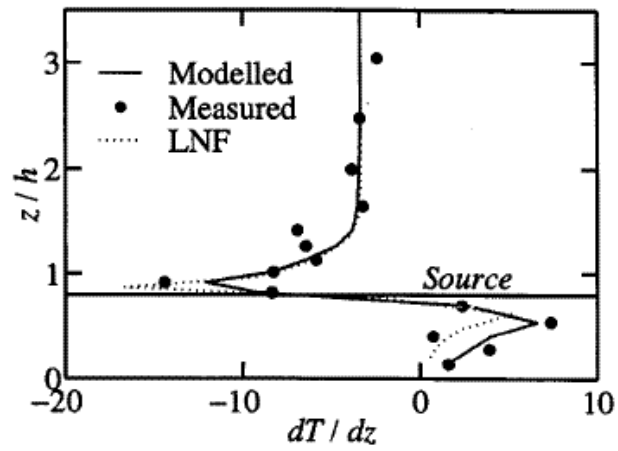


Figure 7. Gradients calculated by the model shown with gradients measured by CRL86. Results from localized near-field theory also shown.

Figure 9 from Warland and Thurtell (2000).

Next Generation

Hybrid Lagrangian/ Eulerian models. Massman and Weil and Katul and students are starting to use Eulerian models to compute turbulent statistics for Lagrangian models, as a means of incorporating aspects of canopy structure, an idea Meyers and I have been discussin for years and are just starting to pursue ourselves.

There is also much effort and advancement in the application of Large Eddy Simulation models into the plant environment. The concepts were first applied to the Atmospheric Boundary layer by Deardorff, (1970) Moeng (1984) and Wyngaard; Schumann, Mason (1994), Andren et al. 1994.. Shaw and Schumann were among the first to apply this method to canopies. Later, students and other colleagues of Shaw (Patton et al; Dwyer et al; Leclerc; Su et al, Albertson) have worked on this method.

Computers are not powerful enough to directly solve the Navier Stokes Equations at the smallest scales (Kolmogorov length scale where viscous dissipation is balanced by shear production). DNS (direct numerical simulation) tends to be applied only for flows with low Reynolds numbers. LES is an intermediate member of the model hierarchy between higher order closure models that solve for time averaged forms of the Navier-Stokes equations and DNS models that resolve all scales.

The LES method performs direct time and space integration of turbulence via the Navier-Stokes equation across a wide spectrum of scales, down to a unresolvable scales, defined by the grid size of the numerical computations. The resolvable grid scale acts as a wave cutoff filter. Smaller scale turbulence is parameterized using eddy diffusivities.

Basic Equations

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2}$$

And the conservation equation

$$\frac{\partial u_i}{\partial x_j} = 0$$

Expanding these equations in terms of resolved and filtered terms leads to Leonard expansion which produces additional covariances. So we can expand u into the resolvable and filtered scales (see Mason, 1994). The filtered scales are averaged over the resolution of the grid.

$$u_i = u_i^r + u_i^f$$

$$\frac{\partial u_i^f}{\partial t} + \frac{\partial u_i^f u_i^f}{\partial x_j} = -\frac{\partial p^f}{\partial x_i} - \frac{\partial (u_i u_j)^f}{\partial x_i} - \frac{\partial u_i^f u_j^f}{\partial x_j}$$

The Leonard rules produce averaging rules similar to Reynolds but for large and small scale components.

$$(u_i u_j)^f = (u_i^f u_j^f)^f + (u_i^r u_j^f)^f + (u_i^f u_j^r)^f + (u_i^r u_j^r)^f$$

So it Typically, scientists represent the canopy/atmosphere domain as a 96 x96x31. To prescribe proper upper boundary conditions, the grid size is about 2 m, still rather large, compared to the resolution of a Lagrangian or Eulerian higher order closure model (eg 40 layers for a 20 m canopy). There are still some computational issues with momentum transfer at the top of the grid, but efforts are being made to remedy such methodological problems.

Albertson et al divided their canopy into 32 by 32 by 100 nodes and covered 100 m vertically and 200 m horizontally. The time step was 1 second. Upper boundary conditions are a problem with LES. Often a zero flux stress free condition is assumed and the horizontal winds are forced with periodic waves of wind. Hence in early papers by Shaw and Schuman the small domain of the canopy did not form a constant flux layer of Reynolds shear stress.

The method is very computationally intense, but it produces turbulence statistics that resemble field measurements. It has lots of potential in my mind to help us understand processes and spatial patterns. Of high importance is the ability of LES models to produce information on pressure fields, pressure fluctuations and gradients (mean and instantaneous). Field data is very ambiguous on this matter, as pressure terms are very hard to measure independent of effects of wind.

The LES TKE budget will have terms for:

- 1) Shear production
- 2) Buoyant production/destruction
- 3) Transport of resolved scale turbulence
- 4) Transport of resolved scale pressure fluctuations
- 5) Work against form drag by velocity perturbations
- 6) Subgrid diffusion of resolved scale TKE
- 7) Transfer of resolved scale TKE to subgrid scales.

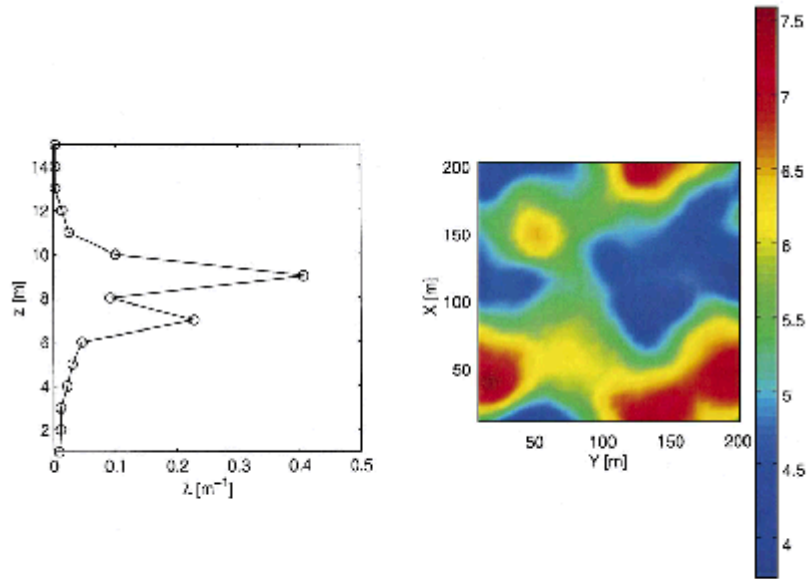


Fig. 1. The left panel shows the normalized vertical shape function of the pine canopy (shape function). The right panel depicts the horizontal distribution of leaf area density used in the heterogeneous canopy simulation.

Figure 10 Albertson et al. LAI distribution as applied to an LES model

Information produced by LES model

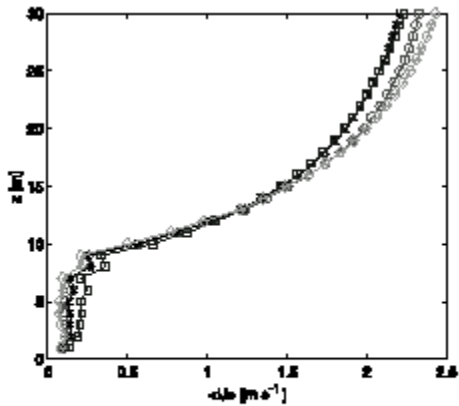


Fig. 2. Mean (horizontal- and time-averaged) profiles of longitudinal wind speed plotted against height in the lower 30 m of the simulation domain. The canopy density shape is distributed vertically according to Fig. 1. The symbols map to the cases as: 'x' = heterogeneous, LAI = 5.5; '□' = homogeneous, LAI = 4; '○' = homogeneous, LAI = 5.5; '◇' = homogeneous, LAI = 7.

Figure 11 Albertson et al

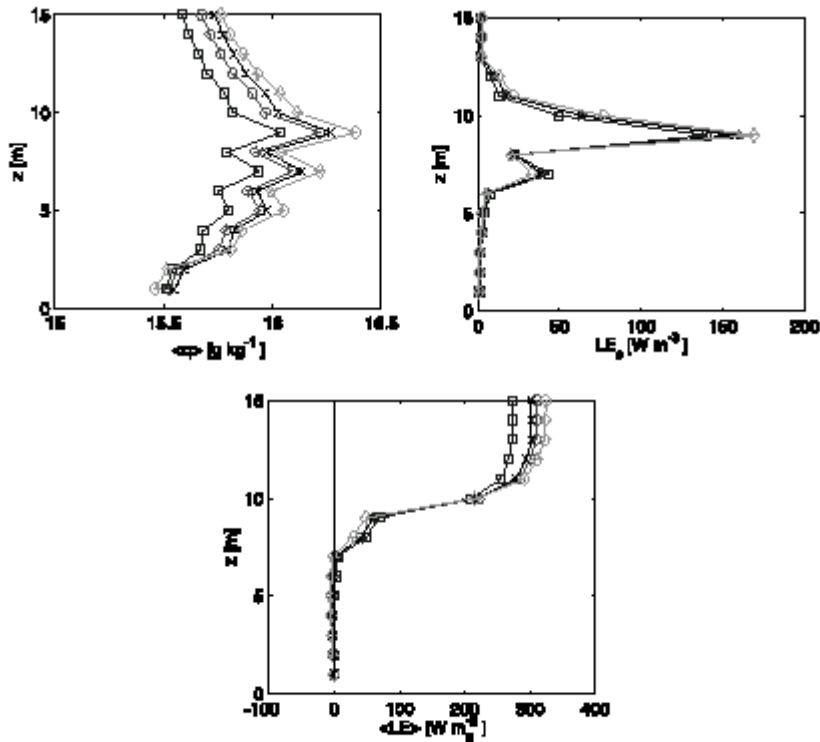


Fig. 4. Mean (horizontal- and time-averaged) profiles related to water vapor exchange and transport. All symbols are as in Fig. 2. The top-left panel shows the profile of water vapor concentration (q) in the air; The top-right panel is the source strength (vegetation's transpiration release) through the canopy. The bottom panel is the mean vertical flux of water vapor carried by the turbulent flow in the canopy space.

Figure 12 Computation of Albertson and Katul

Movies of turbulence in a forest are available on the web at Ned Patton's web site

I encourage you to check out this web site and run the simulations several times and watch the turbulent structures move across the screen. An excellent example of flow visualization.

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